Bonus Culture:

Competitive Pay, Screening, and Multitasking

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Abstract

This paper analyzes the impact of labor market competition and skill-biased technical change on the structure of compensation. The model combines multitasking and screening, embedded into a Hotelling-like framework. Competition for the most talented workers leads to an escalating reliance on performance pay and other high-powered incentives, thereby shifting effort away from less easily contractible tasks such as long-term investments, risk management and within-firm cooperation. Under perfect competition, the resulting efficiency loss can be much larger than that imposed by a single firm or principal, who distorts incentives downward in order to extract rents. More generally, as declining market frictions lead employers to compete more aggressively, the monopsonistic underincentivization of low-skill agents first decreases, then gives way to a growing overincentivization of high-skill ones. Aggregate welfare is thus hill-shaped with respect to the competitiveness of the labor market, while inequality tends to rise monotonically. Bonus caps and income taxes can help restore balance in agents' incentives and behavior, but may generate their own set of distortions.

1 Introduction

Recent years have seen a literal explosion of pay, both in levels and in differentials, at the top echelons of many occupations. Large bonuses and salaries are needed, it is typically said, to retain "talent" and "top performers" in finance, corporations, medicine, academia, as well as to incentivize them to perform to the best of their high abilities. Paradoxically, this trend has been accompanied by mounting revelations of poor actual performance, severe moral hazard and even outright fraud in those same sectors. Oftentimes these behaviors impose negative spillovers on the rest of society (e.g., bank bailouts), but even when not, the firms involved themselves ultimately suffer: large trading losses, declines in stock value, loss of reputation and consumer goodwill, regulatory fines and legal liabilities, or even bankruptcy.

This paper proposes a resolution of the puzzle, by showing how competition for the most productive workers can interact with the incentive structure inside firms to undermine work ethics –the extent to which agents "do the right thing" beyond what their material self-interest commands. More generally, the underlying idea is that highly competitive labor markets make it difficult for employers to strike the proper balance between the benefits and costs of high-powered incentives. The result is a "bonus culture" that takes over the workplace, generating distorted decisions and significant efficiency losses, particularly in the long run. To make this point we develop a model that combines multitasking, screening and imperfect competition, thus making a methodological contribution in the process.

Inside each firm, agents perform both a task that is easily measured (sales, output, trading profits, billable medical procedures) and one that is not and therefore involves an element of public-goods provision (intangible investments affecting long-run value, financial or legal risk-taking, co-operation among individuals or divisions). Agents potentially differ in their productivity for the rewardable task and in their intrinsic willingness to provide the unrewarded one –their work ethic. When types are observable, the standard result applies: principals set relatively low-powered incentives that optimally balance worker's effort allocation; competition then only affects the size of fixed compensation. Things change fundamentally when skill differences are unobservable, leading firms to offer contracts designed to screen different types of workers. A single principal (monopsonist, collusive industry) sets the power of incentives even lower than the social optimum, so as to extract rents from the more productive agents. Labor-market competition, however, introduces a new role for performance pay: because it is differentially attractive to more productive workers, it also serves as a device which firms use to attract (or retain) these types. Focusing first on the limiting case of perfect competition, we show that the degree of incentivization is always above the social optimum, and we identify a simple condition under which the resulting distortion exceeds that occurring under monopsony. Competitive bidding for talent is thus destructive of work ethics, and ultimately welfare-reducing.

We then develop a Hotelling-like variant of competitive screening to analyze the equilibrium contracts under arbitrary degrees of imperfect competition. As mobility costs (or horizontal differentiation) decline, the monopsonistic underincentivization of low-skill agents gradually decreases, then at some point gives way to a growing overincentivization of high-skill ones. Aggregate welfare is thus hill-shaped with respect to competition, while comprehensive measures of inequality (gaps in utility or total earnings) tend to rise monotonically. This leads us to analyze different policies, such as bonus caps or taxes on total compensation, that can potentially improve efficiency and restore balance in agents' incentives and focus. The extent to which this is achievable depends on how well the government or regulator is able to distinguish the incentive versus fixed parts of compensation packages, as well as on the distortions that may arise as firms try to blur that line or resort to even less efficient screening devices.

In our baseline model, one task is unobservable or noncontractible, and thus performed solely out of intrinsic motivation. This (standard) specification of the multitask problem is convenient, but inessential for the main results. We thus extend the analysis to the case where performance in both tasks is measurable and hence rewarded, but noisy, which limits the power of incentives (e.g., deferred compensation versus yearly bonuses) given to risk averse agents. This not only demonstrates robustness (e.g., no reliance on intrinsic motivation) but also yields a new set of results that bring to light how the distorted incentive structure under competition (or monopsony) and the resulting misallocation of effort are shaped by the noise in each task, agents' comparative advantage across them, and risk aversion.

Finally, we contrast our main analysis of competition for talent with the polar case where agents have the same productivity in the measurable task but differ in their ethical motivation for the unmeasurable one. In this case, competition is shown to be either beneficial (reducing the overincentivization which a monopsonist uses to extract rent, but never causing underincentivization), or neutral –as occurs in a variant of the model where ethical motivation generates positive spillovers inside the firm instead of private benefits for the agent.

• *Related evidence.* Although bankers' bonuses and CEO pay packages attract the most attention, the parallel rise in incentive pay and earnings inequality is a much broader phenomenon, as established by Lemieux et al. (2009). Between the late 1970's and the 1990's, the fraction of jobs explicitly paid based on performance rose from 38% to 45%. Further compounding the direct impact on inequality is the fact that the returns to skills, both observable (education, experience, job tenure) and unobservable, are much higher in such jobs. This last finding also suggests that different compensation structures may play an important sorting role.[1] Lemieux et al. calculate that the interaction of structural change and differential returns account for 21% of the growth in the variance of male log-wages over the period, and for essentially 100% (or even more) above the 80^{th} percentile.

The source of escalation in incentive pay in our model is increased competition for the best workers, and this also fits well with the evidence on managerial compensation in advanced countries. In a long-term study (1936-2003) of the market for top US executives, Frydman (2007) documents

[1] Consistent with this view and with our modelling premise that performance incentives affect not only moral hazard (e.g. Bandiera et al. 2007, Shearer 2004) but also selection, Lazear's (2000) study of Safelite Glass Company found that half of the 44% productivity increase reaped when the company replaced the hourly wage system by a piece rate was due to in- and out-selection effects.

a major shift, starting in the 1970's and sharply accelerating since the late 1980's, from firm-specific skills to more general managerial ones –e.g., from engineering degrees to MBA's. In addition, there has been a concomitant rise in the diversity of sectorial experiences acquired over the course of a typical career. Frydman argues that these decreases in mobility costs have intensified competition for managerial skills and shows that, consistent with this view, executives with higher general (multipurpose) human capital received higher compensation and were also the most likely to switch companies. Using panel data on the 500 largest firms in Germany over 1977-2009, Fabbri and Marin (2011) show that domestic and (to a lesser extent) global competition for managers has contributed significantly to the rise of executive pay in that country, particularly in the banking sector.

Our theory is based on competition not simply bidding up the level of compensation at the top, but also significantly altering its structure toward high-powered incentives, with a resulting shift in the mix of tasks performed toward more easily quantifiable and short-term-oriented ones. This seems to be precisely what occurred on Wall Street as market-based compensation spread from the emerging alternative-assets industry to the rest of the financial world:

> "Talent quickly migrated from investment banks to hedge funds and private equity. Investment banks, accustomed to attracting the most-talented executives in the world and paying them handsomely, found themselves losing their best people (and their best MBA recruits) to higher-paid and, for many, more interesting jobs... Observing the remarkable compensation in alternative assets, sensing a significant business opportunity, and having to fight for talent with this emergent industry led banks to venture into proprietary activities in unprecedented ways. From 1998 to 2006 principal and proprietary trading at major investment banks grew from below 20% of revenues to 45%. In a 2006 Investment Dealers' Digest article... one former Morgan Stanley executive said... that extravagant hedge fund compensation –widely envied on Wall Street, according to many bankers– was putting upward pressure on investment banking pay, and that some prop desks were even beginning to give traders "carry." Banks bought hedge funds and private equity funds and launched their own funds, creating new levels of risk within systemically important institutions and new conflicts of interest. By 2007 the transformation of Wall Street was complete. Faced with fierce new rivals for business and talent, investment banks turned into risk takers that compensated their best and brightest with contracts embodying the essence of financial-markets-based compensation." (Desai 2012, *The Incentive Bubble*).

Similar transformations have occurred in the medical world with the rise of for-profit hospital chains: Gawande (2009) documents the escalation of compensation driven by the overuse of revenue-generating tests and surgeries, with parallel declines in preventive care and coordination on cases between specialists, increases in costs and worse patient outcomes.

• *Related literature.* Our paper relates to and extends several lines of work. The first one is that on screening with exclusive contracts, initiated by Rothschild and Stiglitz's (1976) seminal study of a perfectly competitive (free entry) insurance market. Croker and Snow (1985) characterize the Pareto frontier for the two types in the Rothschild-Stiglitz model and show how it ranges from suboptimal insurance for the safer type (as in the original separating equilibrium) to over-insurance for the risky type. Stewart (1994) and Chassagnon and Chiappori (1997) study perfectly competitive insurance markets with both adverse selection and moral hazard: agents can exert risk-reducing efforts, at some privately known cost. In equilibrium the better agents choose contracts with higher deductibles, for which they substitute higher precautionary effort.[2,3] Our paper extends the literature by analyzing screening in a multitask environment and by deriving the equilibrium for the whole range of competition intensities between the polar cases of monopsony and perfect competition, on which most previous work has focused. A notable exception to the latter point is Villas-Boas and Schmidt-Mohr (1999), who study Hotelling competition between banks that screen credit risks through costly collateral requirements. As product differentiation declines they compete more aggressively for the most profitable borrowers, and the resulting increase in screening costs (collateral posted) can be such that overall welfare falls. Banks' problem is one of pure adverse selection, whereas in our context there is also (multidimensional) moral hazard. We thus analyze how the structure of wage contracts, effort allocations, earnings and welfare vary with market frictions. We characterize the socially optimal degree of competitiveness and derive the model's predictions for changes in total pay inequality and its performance-based component, which accord well with the empirical evidence discussed earlier. Stantcheva (2012) studies optimal income taxation when perfectly competitive firms use work hours to screen for workers' productivity. Welfare can then be higher when agents' types are unknown to employers, as the need to signal talent counteracts the Mirrleesian incentive to underproduce. The contrast in results arises from firms and the state being able to observe labor inputs, whereas in our context only output is observable (were it measurable in Stantcheva's single-task model, screening would yield the first best).

From the multitasking literature we borrow and build on the idea that incentivizing easily measurable tasks can jeopardize the provision of effort on less measurable ones (e.g. Holmström and Milgrom 1991, Itoh 1991, Baker at al. 1994, Dewatripont et al. 1999, Fehr and Schmidt 2004). Somewhat surprisingly, the impact of competition on the multitasking problem has not attracted much attention –a fortiori not in combination with adverse selection, which is what generates novel results.[4] As in earlier work, employers choose compensation structures aiming to balance incentives, but the desire to extract rents or the need to select the best employees lead them to offer

[2]Scheuer and Netzer (2010) contrast this beneficial incentive effect of private insurance markets to a benevolent government without commitment power, which would provide full insurance at the interim stage (once efforts have been chosen) and thereby destroy any ex-ante incentive for effort.

[3]Armstrong and Vickers (2001) and Rochet and Stole (2002) study price discrimination in private-value models where, in contrast with the present work, principals do not directly care about agents' types but are purely concerned with rent extraction. Vega and Weyl (2012) study product design when consumer heterogeneity is of high dimension relative to firms' choice variables, which allows for both cream-skimming and rent-extraction to occur in equilibrium.

[4]Acemoglu et al. (2007) show how career concerns can lead workers to engage in excessive signaling to prospective employers, by exerting effort on both a productive task and an unproductive one that makes performance appear better than it really is. Firms could temper career incentives by organizing production according to teamwork, which generates coarser public signals of individual abilities, but the required commitment to team-based compensation fails to be credible when individual performance can still be observed inside the partnership.

socially distorted compensation schemes. In relatively competitive labor markets, in particular, a firm raising its performance-based pay exerts a negative externality: it fails to internalize the fact that competitors, in order to retain their own "talent", will also have to distort their incentive structure and effort allocation, thereby reducing the total surplus generated by their workforce.

While our paper is not specifically about executive pay, this is an important application of the model. The literature on managerial compensation is usually seen as organized along two contrasting lines (see, e.g., Frydman and Jenter (2010) for a recent survey). On one hand is the view that high executive rewards reflect a high demand for rare skills (Rosen 1981) and the efficient workings of a competitive market allocating talent to where it is most productive, for instance to manage larger firms (Gabaix and Landier 2008). Rising pay at the top is then simply the appropriate price response to market trends favoring the best workers: skilled-biased technical change, improvements in monitoring, growth in the size of firms, entry or decreases in mobility costs. On the other side is the view that the level and structure of managerial compensation reflect instead significant market failures. For instance, indolent or captured boards may grant top executives pay packages far in excess of their marginal product, (Bertrand and Mullainathan 2001, Bebchuk and Fried 2004). Alternatively, managers are given incentive schemes that do maximize profits but impose significant negative externalities on the rest of society by inducing excessive short-termism and risk-taking at the expense of consumers, depositors or taxpayers (public bailouts and environmental cleanups, tax arbitrage, etc.) (e.g. Bolton et al. 2006, Besley and Ghatak 2013). In particular, private returns in the finance industry are often argued to exceed social returns (Baumol 1990, Philippon and Reshef 2012). Our paper takes on board the first view's premise that pay levels and differentials largely reflect market returns to both talent and measured performance, magnified in recent decades by technical change and increased mobility. At the same time, and closer in that to the second view, we show that this very same escalation of performance-based pay can be the source of severe distortions and long-run welfare losses in the sectors where it occurs –even absent any externalities on the rest of society, and a fortiori in their presence.

The idea that labor market pressure will force firms to alter the structure of contracts they offer to workers is shared with a couple of recent papers.[5] In Marin and Verdier (2009), international trade integration leads new entrants to compete with incumbents for managerial talent required to operate a firm; within each firm principals also find it increasingly optimal to delegate decision-making to middle-management, further raising the demand for skilled labor. The papers' focus and mechanism are very different from ours, however. Agents receive no monetary incentives but derive private benefits from delegation, and those rents are non-monotonic with respect to competition. Furthermore, equilibrium changes in organizational design and activities performed tend to be efficient responses to relative factor endowments. In Acharya et al. (2011), firms can use two types of incentives: a reward in case of success and the threat of being taken over by a raider in case of failure. The latter deprives the manager of private benefits, so making takeovers hard to resist

[5] There is also an earlier literature examining the (generally ambiguous) effects of *product market* competition on managerial incentives and slack, whether through information revelation (Holmström 1982, Nalebuff and Stglitz 1983) or demand elasticities and the level of profits (Schmidt 1997, Raith 2003).

("strong governance") can be used to economize on bonus pay. Managers with high skills (which are observable) are in short supply, however, so in equilibrium they appropriate all the rents they generate. This forces their employers to renounce the threat of takeovers ("weak governance"), whereas firms employing the more abundant low-skill managers can still avail themselves of it. In contrast to our model, competition weakens here certain aspects of incentives (dismissal for failure) while strengthening others (reward for success). Most importantly, firms' governance choices, and therefore also the competitiveness of the labor market, have no allocative impact: they only redistribute a fixed surplus between managers, shareholders and potential raiders. In Acharya et al. (2012), labor market competition interferes with the process of learning about agents' abilities. A manager can invest in a safe asset or in a risky one whose return depends on his ability but will be observable (to the firm and others) only if he remains in charge of it, with the same employer, for two periods. There are no bonuses, so incentives are implicit (career concerns) ones. Absent mobility, firms can commit ex-ante to paying everyone the same lifetime wage, thus insuring managers against the risk of being of low ability; this also makes it optimal to quickly find out one's talent, so as to choose the type of project one is better suited to. With free mobility, managers who stayed in a firm long enough to be revealed as talented would be bid away by competitors; thus, as in Harris and Holmström (1982), such insurance is precluded. Instead, during the early stages of their careers everyone moves to a different firm in each period so as to delay learning, and in each of these short-term jobs all managers inefficiently select the risky investment, as it has a higher unconditional expected return.

Finally, a recent literature incorporates considerations of intrinsic motivation into compensation design and labor-market sorting. Besley and Ghatak (2005, 2006) find conditions under which agents who derive private benefits from working in mission-oriented sectors will match assortatively with such firms, where they receive low pay but exert substantial effort. Focusing on civil-service jobs, Prendergast (2007) shows how it can be optimal to select employees who are either in empathy with their "clients" (teachers, social workers, firefighters) or somewhat hostile to them (police officers, tax or customs inspectors). When the state has imperfect information about agent's types, however, it is generally not feasible to induce proper self-selection into jobs. Most closely related to our work in this literature is the multitask model of Kosfeld and von Siemens (2011), in which workers differ in their social preferences rather than productivity: some are purely self-interested, others conditional cooperators. Competition among employers leads to agents' sorting themselves between "selfish jobs", which involve high bonuses but no cooperation among coworkers and thus attract only selfish types, and "cooperative jobs" characterized by muted incentives and cooperative behavior, populated by conditional cooperators. Notably, positive profits emerge despite perfect competition. Because the source of heterogeneity is different from the main one emphasized in our paper, it is not surprising that the issue of excessive incentive pay does not arise in theirs.

Section 1 presents the basic model, while Section 2 analyzes the equilibrium under alternative degrees of competition. Section 3 extends the analysis to both tasks being noisily observable and incentivized; it also examines bonus caps and income taxes. Section 4 concludes. The main proofs are gathered in Appendices A to C, more technical ones in supplementary Appendix D.

2 Model

2.1 Agents

• *Preferences.* A unit continuum of agents (workers) engage in two activities, exerting effort a and b respectively:

– Activity A is one in which individual contributions are not (easily) measurable and thus cannot be part of a formal compensation scheme: long-term investments enhancing the firm's value, avoiding excessive risks and liabilities, cooperation, teamwork , etc. An agent's contribution to A is then driven entirely by his intrinsic motivation, va, linear in the effort a exerted in this task. In addition to a genuine preference to "do the right thing" (e.g., an aversion to ripping off shareholders or customers, selling harmful products, teaching shoddily, etc.), v can also reflect social and self-image concerns such as fear of stigma, or an executive's concern for his "legacy".[6] In some contexts it can also capture outside incentives not controlled by the firm, such as potential legal liability.

– Activity B, by contrast, is measurable and therefore contractible: individual output, sales, short-term revenue, etc. When exerting effort b, a worker's productivity is $\theta + b$, where θ is a talent parameter, privately known to each agent.[7]

The total effort cost, $C(a, b)$, is strictly increasing and strictly convex in (a, b), with $C_{ab} > 0$ unless otherwise noted, meaning that the two activities are substitutes. A particularly convenient specification is the quadratic one, $C(a, b) = a^2/2 + b^2/2 + \gamma ab$, where $0 < \gamma < 1$, as it allows for simple and explicit analytical solutions to the whole model.[8] These are provided in Appendix A, whereas in the text we shall maintain a general cost function, except where needed to obtain further results.

We assume an affine compensation scheme with incentive power or bonus rate y and fixed wage z, so that total compensation is $z + (\theta + b)y$.[9] Agents have quasi-linear preferences

$$U(a, b; \theta, y, z) = va + (\theta + b)\, y + z - C(a, b). \tag{1}$$

• *Types.* To emphasize the roles of heterogeneity in v and θ, respectively, we shall focus on two polar cases. Here and throughout Section 3, agents differ only in their productivities. Thus $\theta \in \{\theta_L, \theta_H\}$,

[6] Such preferences leading agents to provide some level of unrewarded effort were part of Milgrom and Holmström' original multitasking model (1991, Section 3). They make the analysis most tractable, but Section 4.2 derives similar results when performance in both tasks is incentivized but A is measured with more noise than B (or/and less discriminating of worker talent). For recent analyzes of intrinsic motivation and social norms see, e.g., Bénabou and Tirole (2003, 2006) and Besley and Ghatak (2005, 2006). In the specific context of firms, see Akerlof and Kranton (2005) for evidence and Ramalingam and Rauh (2010) for a model of investment in employees' loyalty and identification.

[7] The additive form of talent heterogeneity is chosen for analytical simplicity , as it implies that the first best power of incentive is type-independent. Qualitatively similar results would obtain with the multiplicative form $b\theta$, as long as type heterogeneity in θ is not so high that the first-best set of contracts becomes incentive-compatible.

[8] The model also works when the two tasks are complements, $C_{ab} < 0$ (e.g., $-1 < \gamma < 0$) but the results in this case are less interesting, e.g., competition is now, predictably, always more efficient than monopsony.

[9] Unrestricted nonlinear schemes (as in Laffont and Tirole 1986) lead to qualitatively very similar results: see Appendix B and the propositions therein.

with $\Delta\theta \equiv \theta_H - \theta_L > 0$, and θ_i having probability q_i. In Section 4.3, conversely, we shall consider agents who differ only in their intrinsic motivations v for task A.

• *Effort allocation.* When facing compensation scheme (y, z), the agent chooses efforts $a(y)$ and $b(y)$ so as to solve

$$\max_{(a,b)} \{va + (\theta + b)\, y + z - C(a, b)\}, \tag{2}$$

leading to the first-order conditions $\partial C/\partial a = v$, $\partial C/\partial b = y$. Our assumptions on the cost function imply that increasing the power of the incentive scheme raises effort in the measured task and decreases it in the unobserved one:

$$\frac{da}{dy} < \ 0 \ < \frac{db}{dy}.$$

It will prove convenient to decompose the agent's utility into an "allocative" term, $u(y)$, which depends on the endogenous efforts, and a "redistributive" one, $\theta y + z$, which does not:

$$U(y; \theta, z) \equiv U(a(y), b(y); \theta, y, z) = u(y) + \theta y + z, \tag{3}$$

where

$$u(y) \equiv va(y) + yb(y) - C(a(y), b(y)). \tag{4}$$

Note that $u'(y) = b(y)$ and $\partial U(y; \theta, z)/\partial y = \theta + b(y)$.

• *Outside opportunities.* We assume that any agent can obtain a reservation utility $\overline{U}$, so that employers must respect the participation constraint:

$$U(y; \theta, z) = u(y) + \theta y + z \geq \overline{U}. \tag{5}$$

The type-independence of the outside option is a polar case that will help highlight the effects of competition *inside* the labor market. Thus, under monopsony every one has reservation utility equal to $\bar{U}$, whereas with competition reservation utilities become endogenous and type-dependent.[10]

2.2 Firm(s)

A worker of ability θ exerting efforts (a, b) generates a gross revenue $Aa + B(\theta + b)$ for his employer. Employing such an agent under contract (y, z) thus results in a net profit of

$$\Pi(\theta, y, z) = \pi(y) + (B - y)\,\theta - z, \tag{6}$$

where

$$\pi(y) \equiv Aa(y) + (B - y)\, b(y). \tag{7}$$

represents the allocative component and $(B - y)\theta - z$ a purely redistributive one.

[10] We make the usual assumption that when a worker is indifferent between an employer's offer and his reservation utility he chooses the former. We also assume that $\bar{U}$ is high enough that $z \geq 0$ in equilbrium (under any degree of competition), but not so large that hiring some worker types is unprofitable (see Appendix D for the exact conditions).

2.3 Social welfare

In order to better highlight the mechanism at work in the model, we take as our measure of social welfare the sum of workers' and employers' payoffs, thus abstracting from any externalities on the rest of society.[11] Again, it will prove convenient to decompose it into an allocative part, $w(y)$, and a surplus, $B\theta$, that is independent of the compensation scheme (the pure transfer, $(\theta+b)y+z$, nets out):

$$W(\theta, y) \equiv U(a(y), b(y); \theta, y, z) + \Pi(\theta, y, z) = w(y) + B\theta, \tag{8}$$

where

$$w(y) \equiv u(y) + \pi(y) = (A + v)\, a(y) + Bb(y) - C(a(y), b(y)). \tag{9}$$

Using the envelope theorem for the worker, $u'(y) = b(y)$, we have:

$$w'(y) = Aa'(y) + (B - y)\, b'(y). \tag{10}$$

We take w to be strictly concave,[12] with a maximum at $y^* < B$ given by

$$w'(y^*) = Aa'(y^*) + (B - y^*)\, b'(y^*) = 0 \tag{11}$$

and generating enough surplus that even low types can be profitably employed, namely

$$w(y^*) + \theta_L B > \bar{U}. \tag{12}$$

In cases where (underprovision of) the "ethical" activity a also has spillovers on the rest of society –be they technological (pollution), pecuniary (imperfect competition in the product market) or fiscal (cost of government bailouts, taxes or subsidies)– total social welfare becomes $w(y) + e \cdot a(y)$, where e is the per-unit externality. Clearly, this will only strengthen our main results about the competitive overincentivization of the other activity, b.

3 Competing for talent

Throughout most of the paper (except for Section 4.3), v is known while $\theta \in \{\theta_H, \theta_L\}$ is private information, with mean $\overline{\theta} \equiv q_L\theta_L + q_H\theta_H$.[13] We consider in sequence monopsony, perfect and imperfect competition. Before proceeding, it is worth noting that if agents' types $i = H, L$ were

[11] For instance, we can think of firms' output as being sold on a perfectly competitive product market. It is, however, very easy to incorporate social spillovers into the analysis, as we explain below.

[12] Such is the case, in particular, with quadratic costs; see Appendix A.

[13] Asymmetric information about ability remains a concern even in dynamic settings where performance generates ex-post signals about an agent's type. First, such signals may be difficult to accurately observe for employers other than the current one, especially given the multi-task nature of production. Second, many factors can cause θ to vary unpredictably over the life-cycle: age (which affect's people's abilities and preferences heterogeneously), health shocks, private life issues, news interests and priorities, etc. Finally, different (imperfectly correlated) sets of abilities typically become relevant at different stages of a career –e.g., being a good trader or analyst, devising new securities, bringing in clients, closing deals, managing a division, running and growing an international company, etc.

observable, the only impact of market structure would be on the fixed wages z_i , whereas incentives would always remain at the efficient level, $y_i = y^*$.

3.1 Monopsony employer

A monopsonist (or set of colluding firms) selects a menu of contracts (y_i, z_i) aimed at type $i \in \{L, H\}$. We assume that it wants to attract both types, which, as we will show, is equivalent to q_L exceeding some threshold.

The monopsonist maximizes expected profit

$$\max_{\{(y_i,\ z_i)\}_{i=H,L}} \left\{ \sum_{i=H,L} q_i \left[\pi(y_i) + (B - y_i)\theta_i - z_i\right] \right\}$$

subject to the incentive constraints

$$u(y_i) + \theta_i y_i + z_i \geq u(y_j) + \theta_i y_j + z_j \text{ for all } i, j \in \{K, L\} \tag{13}$$

and the low-productivity type's participation constraint

$$u(y_L) + \theta_L y_L + z_L \geq \overline{U}. \tag{14}$$

This program is familiar from the contracting literature. First, incentive constraints, when added up, yield $(\theta_i - \theta_j)(y_i - y_j) \geq 0$, so the power of the incentive scheme must be non-decreasing in type –a more productive agent must receive a higher fraction of his measured output. Second, the low type's incentive constraint is binding, and the high type's rent above $\overline{U}$ is given by the extra utility obtained by mimicking the low type: $(\Delta\theta)y_L$. Rewriting profits, the monopsonist solves:

$$\max_{\{(y_i,\ z_i)\}_{i=H,L}} \left\{ \sum_{i=H,L} q_i \left[w(y_i) + B\theta_i\right] - \overline{U} - q_H(\Delta\theta)y_L, \right\}$$

yielding $y_H^m = y^*$(no distortion at the top) and[14]

$$w'(y_L^m) = \frac{q_H}{q_L}\Delta\theta, \quad \text{implying} \quad y_L < y^*. \tag{15}$$

The principal reduces the power of the low-type's incentive scheme, so as to limit the high-type's rent. It is optimal for the firm to hire both types if and only if

$$q_L \left[w(y_L^m) + B\theta_L - \overline{U}\right] \geq q_H y_L^m \Delta\theta, \tag{16}$$

meaning that the profits earned on low types exceed the rents abandoned to high types. By (15), the difference of the left- and right-hand sides is increasing in q_L, so the condition is equivalent to

[14] To exclude uninteresting corner solutions we shall assume that $w'(0) > q_H \Delta\theta / q_L$. Since later on we shall impose various other upper bounds on q_H, this poses no problem.

$q_L \geq \underline{q}_L$, where $\underline{q}_L$ is defined by equality in (16).

Proposition 1 (monopsony) *Let (16) hold, so that the monopsonist wants to employ both types. Then $y_H^m = y^*$ and $y_L^m < y^*$ is given by*

$$w'(y_L^m) = \frac{q_H}{q_L}\Delta\theta,$$

with corresponding fixed payments $z_H^m = \bar{U} + y_L^m \Delta\theta - u(y^*) - \theta_H y^*$ and $z_L^m = \bar{U} - u(y_L^m) - \theta_L y_L^m$. The resulting welfare loss is equal to

$$L^m = q_L \left[w(y^*) - w(y_L^m) \right]. \tag{17}$$

It increases with $\Delta\theta$, but need not be monotonic in A or B.

Note that since total social welfare is $q_H \left[w(y_H) + B\theta_H \right] + q_L \left[w(y_L) + B\theta_L \right]$, a mean-preserving increase in the distribution of θ always reduces it, by worsening the informational asymmetry. In contrast, an increase in A (or a decrease in B) has two opposing effects on L^m : (i) it makes any given amount of underincentivization on the B task less costly, as the alternative task A is now more valuable; (ii) the efficient bonus rate y^* given to the high types declines, and to preserve incentive compatibility so must y_L^m, worsening low types' underincentivization. In the quadratic case the two effects cancel out, as shown in Appendix A.

3.2 Perfect competition in the labor market

A large number of firms now compete for workers, each one offering an incentive-compatible menu of contracts. We first look for a separating competitive allocation, defined as one in which: (i) each worker type chooses a different contract, respectively (y_L, z_L) and (y_H, z_H) for $i = H, L$, with resulting utilities U_L and U_H; (ii) each of these two contracts makes zero profits, implying in particular the absence of any cross-subsidy.[15] Then, in a second stage, we investigate the conditions under which this allocation is indeed an equilibrium, and even the unique one.

In a separating competitive equilibrium, any contract that operates must make zero profit:

$$\Pi(\theta_H, y_H, z_H) = 0 \iff \pi(y_H) + (B - y_H)\,\theta_H = z_H, \tag{18}$$

$$\Pi(\theta_L, y_L, z_L) = 0 \iff \pi(y_L) + (B - y_L)\,\theta_L = z_L, \tag{19}$$

which pins down z_H and z_L. Furthermore, a simple Bertrand-like argument implies that the low type must receive his symmetric-information efficient allocation,[16]

[15] One can then can indifferently think of each firm offering a menu and employing both types of workers, or of different firms specializing in a single type by offering a unique contract.

[16] Absent cross-subsidies, the low type cannot receive more than the total surplus $w(y^*) + \theta_L B$ he generates under symmetric information, or else his employer would make a negative profit. Were he to receive less, conversely, another firm could attract him by offering $(y^*, z_L = z_L^c - \varepsilon)$ for ε small, leading to a profit ε on this type (and an even larger one on any high type who also chose this contract). Low types must thus be offered utility equal to $w(y^*) + \theta_L B$, which only their symmetric-information efficient allocation achieves.

$$y_L^c = y^* \quad \text{and} \quad z_L^c = \pi(y^*) + (B - y^*)\theta_L.$$

He should then not benefit from mimicking the high type, nor vice-versa,

$$w(y^*) + B\theta_L \quad \geq \quad u(y_H) + \theta_L y_H + z_H = w(y_H) + B\theta_H - y_H \Delta\theta, \tag{20}$$

$$w(y_H) + B\theta_H \quad \geq \quad w(y^*) + B\theta_L + y^* \Delta\theta, \tag{21}$$

implying in particular that $y_H \geq y^*$. Among all such contracts, the most attractive to the high types is the one involving minimal distortion, namely such that (20) is an equality:

$$w(y_H^c) = w(y^*) - (B - y_H^c)\,\Delta\theta. \tag{22}$$

By strict concavity of w, this equation has a unique solution y_H^c to the right of y^*, satisfying $y^* < y_H^c < B$. The inequality in (20) is then strict, meaning that only the low type's incentive constraint is binding. Note that, as illustrated in Figure I, this is exactly the reverse of what occurred under monopsony.[17]

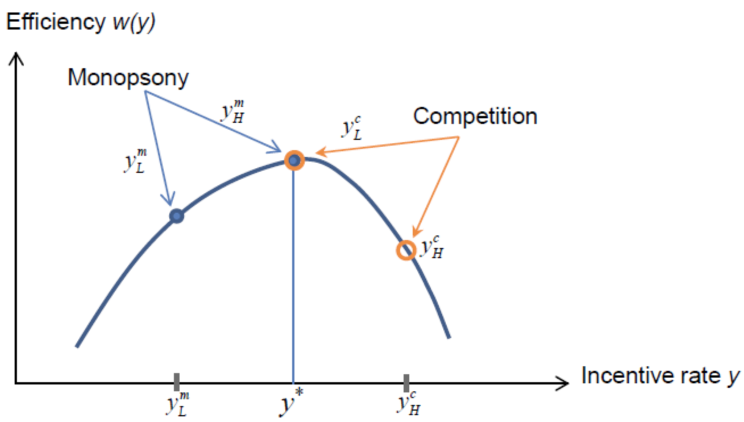

Figure I: Distortions under monopsony and perfect competition

• *Existence and uniqueness.* When is this least-cost separating (LCS) allocation indeed an equilibrium, or the unique equilibrium of the competitive-offer game? The answer, which is reminiscent of Rothschild and Stiglitz (1976), hinges on whether or not a firm could profitably deviate to a contract that achieves greater total surplus by using a cross-subsidy from high to low types to

[17] To build intuition for this reversal, one can look at how the symmetric-information outcome fails incentive compatibility. Under monopsony no employee obtains any rent, so type i's symmetric information contract is: $(y_i = y^*,\ z_i = \bar{U} - va(y^*) + C(a(y^*),\ b(y^*)) - [\theta_i + b(y^*)]y^*)$. If it were still offered under asymmetric information, the high type could then obtain a positive rent $(\Delta\theta)y^*$ by choosing (y^*, z_L); we thus expect the downward incentive constraint to bind. Under perfect competition, the symmetric-information contract for type i is $(y_i = y^*,\ z_i = \pi(y^*) + (\theta_i - y^*)B)$. If it prevailed under asymmetric information, the low type could obtain extra utility $B\Delta\theta$ by choosing $(y_H,\ z_H)$. We thus now expect the upward incentive constraint to bind.

ensure incentive compatibility.

Definition 1 *An incentive-compatible allocation* $\{(U_i^*, y_i^*)\}_{i=H,L}$ *is interim efficient if there exists no other incentive-compatible* $\{(U_i, y_i)\}_{i=H,L}$ *that*
(i) Pareto dominates it: $U_H \geq U_H^*$ *,* $U_L \geq U_L^*$*, with at least one strict inequality.*
(ii) Makes the employer break even on average: $\Sigma_i q_i[w(y_i) + \theta_i B - U_i] \geq 0$.

For the LCS allocation to be an equilibrium, it must be interim efficient. Otherwise, there is another menu of contracts that Pareto dominates it, which one can always slightly modify (while preserving incentive-compatibility) so that both types of workers and the employer share in the overall gain; offering such a menu then yields strictly positive profits. The converse result is also true: under interim efficiency there can clearly be no positive-profit deviation that attracts both types of agents, and by a similar type of "small redistribution" argument one can also exclude those that attract a single type. These claims are formally proved in the appendix, where we also show that when the LCS allocation is interim efficient, it is in fact the unique equilibrium. Furthermore, we identify a simple condition for this to be case:

Lemma 1 *The least-cost separating allocation is interim efficient if and only if*

$$q_H w'(y_H^c) + q_L \Delta\theta \geq 0. \tag{23}$$

This condition holds whenever q_L *exceeds some threshold* $\tilde{q}_L < 1$.

The intuition is as follows. At the LCS allocation, we saw that the binding incentive constraint is the low type's: $U_L^c = U_H^c - y_H^c \Delta\theta$. Consider now an employer who slightly reduces the power of the high type's incentive scheme, $\delta y_H = -\varepsilon$, while using lump-sum transfers $\delta z_H = (b(y_H) + \theta_H)\varepsilon + \varepsilon^2$ to slightly more than compensate them for the reduction in incentive pay, and $\delta z_L = \varepsilon\Delta\theta + \varepsilon^2$ to preserve incentive compatibility. Such a deviation attracts both types ($\delta U_H = \delta U_L = \varepsilon^2$, since $u'(y) = b(y)$) and its first-order impact on profits is

$$q_H \left[\left(\pi'(y_H) - \theta_H \right) \delta y_H - \delta z_H \right] - q_L \delta z_L = \left[q_H \ w'(y_H^c) + q_L \Delta\theta \right] (-\varepsilon)$$

Under (23) this net effect is strictly negative, hence the deviation unprofitable. When (23) fails, conversely, the increase in surplus generated by the more efficient effort allocation of the high types is sufficient to make the firm and all its employees strictly better off. A higher $q_L = 1 - q_H$ means fewer high types to generate such a surplus and more low types to whom rents (cross-subsidies) must be given to maintain incentive compatibility, thus making (23) more likely to hold.

We can now state this section's main results.

Proposition 2 (perfect competition) *Let* $q_L \geq \tilde{q}_L$*. The unique competitive equilibrium involves two separating contracts, both resulting in zero profit:*

1. *Low-productivity workers get* (y^*, z_L^c)*, where* z_L^c *is given by (19).*

2. High-productivity ones get (y_H^c, z_H^c)*, where* z_H^c *is given by (18) and* $y_H^c > y^*$ *by*

$$w(y^*) - w(y_H^c) = (B - y_H^c)\Delta\theta.$$

3. The efficiency loss relative to the social optimum is

$$L^c = q_H\left[w(y^*) - w(y_H^c)\right] = (B - y_H^c)\, q_H \Delta\theta. \tag{24}$$

It increases with $\Delta\theta$ *and* A*, but need not be monotonic in* B*.*

These results confirm and formalize the initial intuition that competition for talent will result in an overincentivization of higher-ability types. As shown on Figure I, this is the opposite distortion from that of the monopsony case, which featured underincentivization of low-ability types. When the degree of competition is allowed to vary continuously (Section 3.4), we therefore expect that there will be a critical point at which the nature of the distortion (reflecting which incentive constraint is binding) tips from one case to the other.

• *Skill-biased technical change.* A higher θ_H exacerbates the competition for talented types, resulting in a higher bonus rate y_H^c that makes their performance-based pay rise *more than proportionately* with their productivity $B\theta_H$. This is in line with Lemieux et al.'s (2009)) findings about the contribution of performance pay to rising earnings inequality. This equilibrium market response to technical or human-capital change is *inefficient,* however, as it worsens the underprovision of long-term investments and other voluntary efforts inside firms, thereby reducing the social value of the productivity increase. For a mean-preserving spread in the distribution of θ's only the deadweight loss remains, so overall social welfare declines.

What happens when the LCS allocation is not interim efficient, that is, when $q_L < \tilde{q}_L$? We saw that it is then not an equilibrium, since there exist profitable deviations to incentive-compatible contracts (involving cross-subsidies) that Pareto-dominate it. We also show in the appendix that no other pure-strategy allocation is immune to deviations, a situation that closely parallels the standard Rothschild and Stiglitz (1976) problem: the only equilibria are in mixed strategy.[18] Since such an outcome is not really plausible as a stable labor-market outcome, we assume from here on

$$q_L \geq \max\{\tilde{q}_L, \underline{q}_L\} \equiv q_L^*. \tag{25}$$

[18] An alternative approach is to assume that it is workers who make take-it-or-leave offers, instead of a competitive industry making offers to them. From Maskin and Tirole (1992) we know that for $q_L \geq \tilde{q}_L$, the unique equilibrium of the resulting informed-principal game is the LCS allocation, so the result is the same as here with competitive offers. By contrast, for $q_L \geq \tilde{q}_L$, the set of equilibrium interim utilities is the set of feasible utilities (incentive compatible and satisfying budget balance in expectation) that Pareto dominate (U_L^c, U_H^c). A second alternative is to use a different equilibrium concept from the competitive screening literature, as in Scheuer and Netzer (2010); again, this has no bearing on the region where the separating equilibrium exists. A third alternative would be to introduce search, free entry by principals and contract posting as in Guerrieri et al. (2010). Self-selection then makes type proportions among searchers in the market endogenous, in such a way that a separating equilibrium always exists.

3.3 Welfare: monopsony versus perfect competition

• *Single task* ($A = 0$). As a benchmark, it is useful to recall that competition is always socially optimal with a single task. The competitive outcome is then the single contract $y^c = y^* = B,\ z^c = 0$; agents of either type are residual claimants for their production and therefore choose the efficient effort allocation.[19] Monopsony, by contrast, leads to a downward distortion in the power of the incentive scheme. Hence competition is always strictly welfare superior.

• *Multitasking.* From (17) and (22), $L^m < L^c$ is larger if and only if

$$q_L \left[w(y^*) - w(y_L^m)\right] < q_H \left[w(y^*) - w(y_H^c)\right]. \tag{26}$$

Consider first the role of labor force composition. As seen from (15) and (22), the monopsony incentive distortion $y^* - y_L^m$ is increasing with q_H/q_L (limiting the high types's rents becomes more important), whereas the competitive one, $y_H^c - y^*$ is independent of it (being determined by an incentive constraint across types). For small q_H/q_L, L^m/L^c is thus of order $q_L \left(q_H/q_L\right)^2/q_H = q_H/q_L$, so (26) holds provided q_L is high enough. With quadratic costs, we obtain an exact threshold that brings to light the role of the other forces at play.

Proposition 3 (welfare) *Let $C(a,b) = a^2/2 + b^2/2 + \gamma ab$. Social welfare is lower under competition than under monopsony if and only if $q_L \geq q_L^*$ and*

$$\frac{q_H}{2q_L} + \sqrt{\frac{q_H}{q_L}} < \left(\frac{\gamma}{1-\gamma^2}\right)\left(\frac{A}{\Delta\theta}\right). \tag{27}$$

The underlying intuitions are quite general.[20] First, competition entails a larger efficiency loss when the unrewarded task (long-run investments, cooperation, avoidance of excessive risks, etc.) is important enough and the two types of effort sufficiently substitutable. If they are complements ($\gamma < 0$), in contrast, competition is always efficiency-promoting. Second, the productivity differential $\Delta\theta$ scales the severity of the asymmetric-information problem that underlies both the monopsony and the competitive distortions. A monopsonistic firm optimally trades off total surplus versus rent-extraction, so (by the envelope theorem) a small $\Delta\theta$ has only a second-order effect on overall efficiency. Under competition the effect is first-order, because a firm raising its y_H does not internalize the deterioration in the workforce quality it inflicts on its competitors –or, equivalently, the fact that in order to retain their "talent" they will also have to distort incentives and the allocation of effort. This intuition explains why (27) is more likely to hold when $\Delta\theta$ decreases.[21]

[19] Similar results holds if $A > 0$ but $v = 0$: since $a \equiv 0$ for all y, the socially optimal bonus rate is $y^* = B$, even though it results in an inefficient effort allocation. This is clearly also the competitive outcome. .

[20] In particular, the model's solution with quadratic costs (given in Appendix A) also correspond to Taylor approximations of the more general case when $\Delta\theta$ is small, provided $1/(1-\gamma^2)$ is replaced everywhere by $-w''(y^*)$.

[21] As shown in Appendix C, for small $\Delta\theta$ the lower bound $\tilde{q}_L$ above which (23) holds (and the competitive equilibrium thus exists) is such that $1 - \tilde{q}_L$ is of order $\sqrt{\Delta\theta}$. Thus, to rigorously apply the above reasoning involving first- versus second-order losses for small $\Delta\theta$, one needs to also let q_H become small. This further reduces $y^* - y_L^m$ while leaving $y_H^c - y^*$ unchanged. This, in turn, further raises L^c/L^m, making it of order $(\Delta\theta)^{-3/2}$ rather than $(\Delta\theta)^{-1}$.

3.4 Imperfect competition

To understand more generally how the intensity of labor market competition affects the equilibrium structure of wages, workers' task allocation, firms' profits and social welfare, we now develop a variant of the Hotelling model in which competitiveness can be appropriately parametrized.

As illustrated in Figure II, a unit continuum of workers is uniformly distributed along the unit interval, $x \in [0,1]$. Two firms, $k = 0, 1$, are located respectively at the left and right extremities and recruit them to produce, with the same production function as before. When a worker located at x chooses to work for Firm 0 (resp., 1), he incurs a cost equal to the distance tx (resp., $t(1-x)$) that he must travel. We assume that θ and x are independent and that a worker's position is not observable by employers, who therefore cannot condition contracts on this characteristic.

In the standard Hotelling model, agents also have an outside option (e.g., staying put) that yields a fixed level of utility, $\bar{U}$. This implies, however, that a change in t affects not only competitiveness *within* the market (firm 1 vs. 2) but also, mechanically, that of the *outside* option –formally, agents' participation constraints. To isolate the pure competitiveness of the market from that of other activities, we introduce an intuitive but novel modeling device, ensuring in particular that the market is always fully covered.

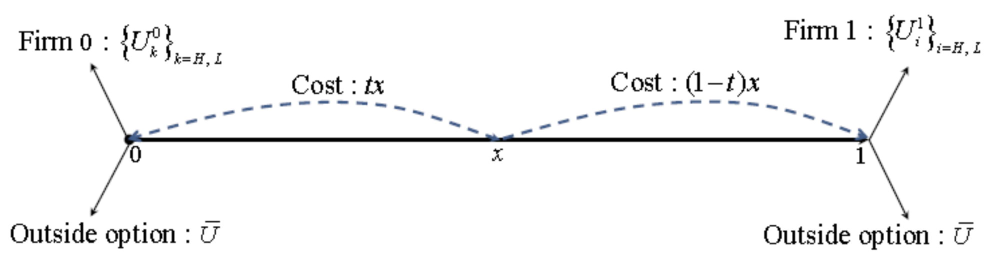

Figure II: Hotelling with co-located outside options

• *Co-located outside option.* Instead of receiving the outside option $\bar{U}$ for free, agents must also "go and get it" at either the end of the unit interval, which involves paying the same cost tx or $t(1-x)$ as if they chose Firm 0 or Firm 1, respectively. One can think of two business districts, each containing both a multitask firm of the type studied here and a competitive fringe or informal sector in which all agents have productivity $\bar{U}$. Alternatively, each agent could produce $\bar{U}$ "at home" but then have to travel (or adapt) to one or the other marketplace to sell his output.

Without loss of generality, we can assume that each firm $k = 0, 1$ offers an incentive-compatible menu of compensation schemes$\{y_i^k, z_i^k\}_{i=H,L}$, in which workers who opt for this employer self-select. Let U_i^k denote the utility provided by firm k to type i:

$$U_i^k \equiv u(y_i^k) + \theta_i y_i^k + z_i^k. \tag{28}$$

A worker of type i, located at x, will choose firm $k = 0$ (say) if and only if

$$U_i^k - tx \geq \max\left\{\bar{U} - tx, \bar{U} - t(1-x), U_i^\ell - t(1-x)\right\}. \tag{29}$$

The first inequality reduces to $U_i^k \geq \bar{U}$: a firm must at least match its local outside option. If both firms attract L-type workers, therefore, $U_i^\ell \geq \bar{U}$ so the second inequality is redundant.

We shall focus the analysis on the (unique) symmetric equilibrium, in which firms attracts half of the total labor force. To simplify the exposition, we shall take it here as given that: (i) each firm prefers to employ positive measures of both types of workers than to exclude either one; (ii) conversely, neither firm wants to "corner" the market on any type of worker, i.e. move the corresponding cutoff value of x all the way to 0 or 1. In Appendix D we show that neither exclusion nor cornering can be part of a best response by a firm to its competitor playing the strategy characterized in Proposition 4 below, as long as

$$q_L \geq \bar{q}_L, \tag{30}$$

where $\bar{q}_L \in [q_L^*, 1)$ is another cutoff independent of t. Assuming this condition from here on, we can focus on utilities $\left(U_i^k, U_i^\ell\right)$ resulting in interior cutoffs, so that firm k's share of workers of type i is

$$x_i^k\left(U_i^k, U_i^\ell\right) = \frac{U_i^k - U_i^\ell + t}{2t}. \tag{31}$$

The firm then chooses (U_L, U_H, y_L, y_H) to solve the program:

$$\max\left\{q_H(U_H - U_H^\ell + t)[w(y_H) + \theta_H B - U_H] + q_L(U_L - U_L^\ell + t)[w(y_L) + \theta_L B - U_L]\right\} \tag{32}$$

subject to:

$$\begin{aligned} U_H &\geq U_L + y_L\Delta\theta \qquad (\mu_H) && (33)\\ U_L &\geq U_H - y_H\Delta\theta \qquad (\mu_L) && (34)\\ U_L &\geq \bar{U} \qquad\qquad\quad (\nu) && (35)\end{aligned}$$

To shorten the notation, let $m_i \equiv w(y_i) + \theta_i B - U_i$ denote the firm's margin on type $i = H, L$. The first-order conditions, together with the requirement that $U_i = U_i^\ell$ in a symmetric equilibrium, are:

$$\begin{aligned} q_H(m_H - t) + \mu_H - \mu_L &= 0 && (36)\\ q_L(m_L - t) + \mu_L - \mu_H + \nu &= 0 && (37)\\ tq_H w'(y_H) + \mu_L\Delta\theta &= 0 && (38)\\ tq_L w'(y_L) - \mu_H\Delta\theta &= 0. && (39)\end{aligned}$$

Note that μ_H and μ_L cannot both be strictly positive: otherwise (33) and (34) would bind, hence $y_H = y_L$, rendering (38)-(39) mutually incompatible. This suggests that only one or the other incentive constraint will typically bind at a given point.

• *Constructing the equilibrium: key intuitions.* Solving the above problem over all values of t is quite complicated, so we shall focus here on the underlying intuitions. The solution to (33)-(39) is formally derived in Appendix C; because the objective function (32) is not concave on the relevant space for (U_L, U_H, y_L, y_H), Appendix D then provides a constructive proof that this allocation is indeed the global optimum. These and other technical complexities (exclusion, cornering) are the reasons why we confine our analysis to the symmetric separating equilibrium.

(a) For large t, the equilibrium should resemble the monopsonistic one: the main concern is limiting high types' rent, so firms distort $y_L < y^* = y_H$ to make imitating low types unattractive. Conversely, for small t, the equilibrium should resemble perfect competition: the main concern is attracting the H types, leading employers to offer them high-powered incentives, $y_H > y^* = y_L$.

(b) As t declines over the whole real line, the high types' responsiveness to higher offered utility U_H rises, so firms are forced to leave them more rent. Since that rent is either $y_L\Delta\theta$ or $y_H\Delta\theta$ (depending on which of the above two concerns dominates, i.e. on which types' incentive constraint is binding), y_L and y_H should both be nonincreasing in t.

(c) Firms 0 and 1 are always actively competing for the high types. If t is low enough, they also compete for L types, offering them a surplus above their outside option: $U_L > \bar{U}$. At the threshold t_1 below which U_L starts exceeding $\bar{U}$, y_H has a convex kink: since the purpose of keeping y_H above y^* is to maintain a gap $U_H - U_L = y_H\Delta\theta$ just sufficient to dissuade low types from imitating high ones, as U_L begins to rise above $\bar{U}$, the rate of increase in y_H can be smaller.

These intuitions translate into a characterization of the equilibrium in terms of three regions, illustrated in Figure III and formally stated in Proposition 4 below.[22]

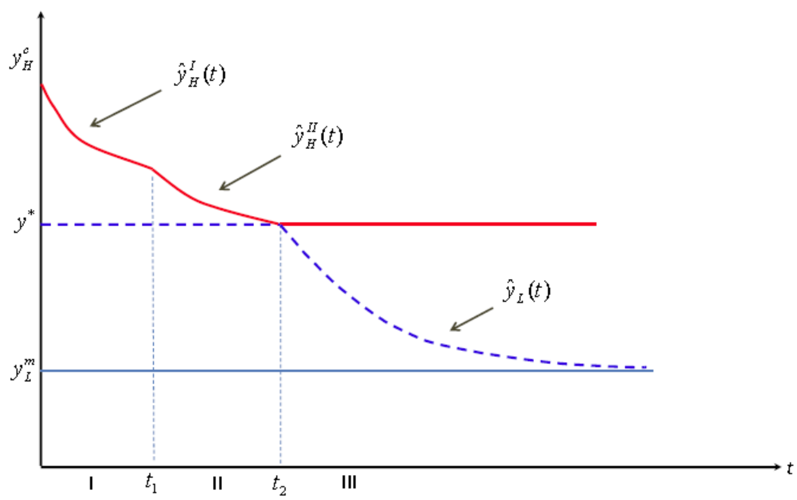

Figure III: equilibrium incentives under imperfect competition

Proposition 4 (imperfect competition) *Let $q_L \geq \bar{q}_L$. There exist unique thresholds $t_1 > 0$ and $t_2 > t_1$ such that, in the unique symmetric market equilibrium:*

1. *Region I (strong competition): for all $t < t_1$, bonuses are $y_L = y^* < \hat{y}_H^I(t)$, strictly decreasing in t, starting from $\hat{y}_H^I(0) = y_H^c$. The low type's participation constraint is not binding, U_L*

[22]With quadratic costs one can show (see Appendix A) that each of the curves is convex, as drawn on the figure.

$> \bar{U}$, *while his incentive constraint constraint is:* $U_H - U_L = \hat{y}_H^I(t)\Delta\theta$.

2. *Region II (medium competition): for all* $t \in [t_1, t_2)$, *bonuses are* $y_L = y^* < \hat{y}_H^{II}(t)$, *with* $\hat{y}_H^{II}(t) < \hat{y}_H^I(t)$ *except at* t_1 *and strictly decreasing in* t. *The low type's participation constraint is binding,* $U_L = \bar{U}$, *and so is his incentive constraint:* $U_H - U_L = \hat{y}_H^{II}(t)\Delta\theta$.

3. *Region III (weak competition): for all* $t \geq t_2$, *bonuses are* $y_L = \hat{y}_L(t) < y^* = y_H$, *with* $\hat{y}_L(t)$ *strictly decreasing in* t *and* $\lim_{t\to+\infty} \hat{y}_L(t) = y_L^m$. *The low type's participation and the high type's incentive constraints are binding :* $U_L = \bar{U}$, $U_H - U_L = \hat{y}_L(t)\Delta\theta$.

• *Welfare.* For each value of t, either y_L or y_H is equal to the (common) first-best value y^*, while the other bonus rate, which creates the distortion, is strictly increasing in t. Recalling from (8)-(9) that $W = q_H w(y_H) + q_L w(y_L) + B\bar{\theta}$ we thus have, as illustrated on Figure IV:

Proposition 5 (optimal degree of competition) *Social welfare is hill-shaped as a function of the degree of competition in the labor market, reaching the first-best at* $t_2 = w(y^*) + \theta_H(B - y^*) + \theta_L y$, *where* $y_L = y^* = y_H$.

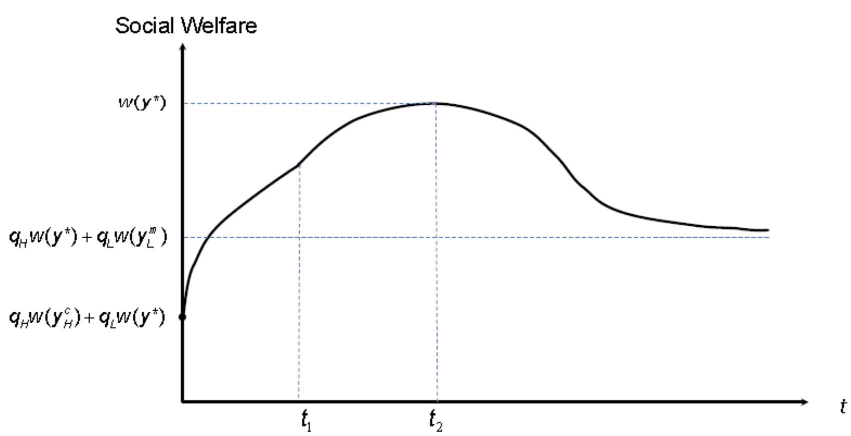

Figure IV: competition and social welfare

Note that we do not subtract from W the total mobility cost $t/4$ incurred by agents (equivalently, we add it to their baseline utility). This is consistent with using t as a measure of pure market competitiveness, without introducing any additional effect. In particular, it is *required* to yield back the monopsony levels of utility as $t \to +\infty$. One can think of t as a tax on mobility rebated to agents, or as the profits of a monopolistic transportation or human-capital-adaptation sector with zero marginal cost, engaged in limit pricing against a competitive fringe with marginal cost t. Alternatively, in contexts where variations in t also involve a net resource cost, one could subtract it from social welfare (as in Villas-Boas and Schmidt-Mohr 1999). In Appendix A we show that increases in t can raise aggregate welfare even under this more demanding definition.

We next examine how the gains and losses in total welfare (under either definition) are distributed among the different actors in the market.

Proposition 6 (individual welfare and firm profits) *As the labor market becomes more competitive (t declines), both U_H and U_L increase (weakly for the latter), but inequality in workers' utilities, $U_H - U_L$ always strictly increases; firms' total profits strictly decline.*

In Regions III and II, $U_L = \bar{U}$. In Region I, U_L is decreasing in t, as we show in the appendix. Since $(U_H - U_L)/\Delta\theta$ is equal to $\hat{y}_H(t)$ over Regions I and II and to $\hat{y}_L(t)$ over Region III, it follows directly from Proposition 4 that $\partial U_H/\partial t \leq \partial (U_H - U_L)/\partial t < 0$. As to profits, they must clearly fall as t declines over Regions II and I, since overall surplus is shrinking but all workers are gaining. In Region III, as $\hat{y}_L(t)$ rises firms reap some of the efficiency gains from low-type agent's more efficient effort allocation, but the rents they must leave to high types increase even faster (as shown in the appendix), so total profits decline here as well.

• *Income inequality.* We now consider the effects of a more competitive labor market on earnings, which is what is measured in practice. While we analyze these comparative statics over all $t \in \mathbb{R}_+$, the empirically relevant range for most sectors in a modern market economy is that of *medium to high mobility*, namely Regions I and II. Indeed, this is where firms are more concerned with retaining and bidding away from each other the high-ability types who can easily switch (x close to 1/2) than with exploiting their more "captive" local markets (forcing down the rents of those with x close to 0 or 1). We compare how the two types of workers fare in terms of total earnings $Y_i \equiv [b(y_i) + \theta_i]\, y_i + z_i$, as well as the separate contributions of performance-based and fixed pay.

Proposition 7 (income inequality) *Let $q_L \geq \bar{q}_L$. As the labor market becomes more competitive (t declines), both Y_H and Y_L increase (weakly for the latter). Furthermore,*

1. *Over Regions I and II (medium and high competition), inequality in total pay $Y_H - Y_L$ rises, as does its performance-based component. Inequality in fixed wages declines, so changes in performance pay account for more than 100% of the rise in total inequality.*

2. *Over Region III (low competition), inequality in performance pay declines, while inequality in fixed wages rises. As a result, inequality in total pay need not be monotonic. With quadratic costs, a sufficient condition for it to rise as t declines is $B \leq \gamma A + (1 - \gamma^2)\Delta\theta$.*

These results are broadly consistent with the findings of Lemieux et al. (2009) about the driving role of performance pay in rising earnings inequality, as well as their hypothesis that the increased recourse to performance pay also serves a screening purpose. They are also in line with Frydman's (2007) evidence linking increased mobility (skills portability) of corporate executives to the rise in both the level and the variance in their compensation.[23]

We demonstrate here the results for total earnings, leaving the others to the appendix. Since $z_i = U_i - u(y_i) - \theta_i y_i$, we can write $Y_i = U_i + b(y_i)y_i - u(y_i)$, for $i = H, L$. As t declines, U_i and y_i increase (at least weakly) and therefore so does Y_i, since $u'(y) = b$. Furthermore,

$$Y_H - Y_L = U_H - U_L + b(y_H)y_H - u(y_H) + u(y_L) - b(y_L)y_L.$$

[23] See also Frydman and Saks' (2005) evidence on the rising share of performance pay in compensation.

Over Regions I and II this becomes $[\Delta\theta + b(y_H)]\, y_H - u(y_H)$ plus a constant term, with $y_H = \hat{y}_H(t)$; the result then follow from $u'(y) = b$. Over Region III, $Y_H - Y_L = [\Delta\theta - b(y_L)]\, y_L + u(y_L)$ plus a constant term, with $y_L = \hat{y}_L(t)$; therefore, $\partial(Y_H - Y_L)/\partial t \; < 0$ if and only if $b'(y_L)y_L < \Delta\theta$, which need not hold in general. With quadratic costs, $b'(y_L) = 1/(1-\gamma^2)$ so it holds on $[t_2, +\infty)$ if and only if $y^* = B - \gamma A < (1-\gamma^2)\Delta\theta$.

4 Extensions

4.1 Regulating compensation

Recent years have seen mounting pressure from the public, regulators and even shareholders to limit the bonuses paid in the financial sector and similar high-powered incentives (e.g., stock and options grants) given to top executives in other industries. We examine this issue in light of our model, focusing on the case of perfect competition as it is both the most empirically relevant and that in which the efficiency losses from the "bonus culture", such as excessive short-term focus or risk-taking, are most severe.[24] A first question is whether the government or regulator is able to distinguish and treat differently the performance-related and fixed parts of compensation. If it is, then absent any other decision margin that could be distorted, policy can be very effective.

Proposition 8 (efficient bonus cap) *If the regulator caps bonuses at y^*, the only equilibrium is a pooling one in which all firms offer, and all workers take, the single contract $(y^*, \pi(y^*) + (B - y^*)\bar{\theta})$, thereby restoring the first best.*

Of course, things in practice may not be so simple. First, firms may relabel fixed and variable compensation, in which case only total pay can be regulated, or taxed. Second, they may switch to alternative forms of rewards that (at the margin) appeal differentially to different types but may be even less efficient screening devices than performance bonuses. Plausible examples include latitude to serve on other companies' boards, to engage in own practice (doctors) or consulting (academics), lower lock-in to company (low clawbacks, easier terms for quitting). To what extent each of these is indeed more valued (relative to money) by more talented agents, and how the resulting distortions compare to those arising from excessive bonuses, are ultimately empirical questions. Our purpose in what follows is simply to show how, when such inefficient screening devices are readily available, pay regulation can also backfire.

Suppose that \$1 paid by the employer in the alternative "currency" yields utility $\$\lambda_i$ to a type-i employee, where $\lambda_L < \lambda_H < 1$. Assume that in the absence of regulation, employers do not make use of the inefficient transfer to screen workers:

$$\frac{|w'(y_H^c)|}{\Delta\theta} < \frac{1-\lambda_H}{\Delta\lambda}, \tag{40}$$

[24] Besley and Ghatak (2013) explore a different rationale, based on the negative externalities arising from government bailouts, for taxing the bonuses of financial intermediaries. In contrast, we continue to focus here on the benchmark case where workers' and firms' activities have no spillovers on the rest of society. When the "ethical" activity A (or its underprovision) does have such effects, $e \cdot a(y)$, the case for regulating compensation is naturally strengthened.

where $\Delta\lambda \equiv \lambda_H - \lambda_L$. The left-hand side measures the marginal distortion avoided by decreasing the monetary bonus by \$1/Δ, which raises the low type's utility from mimicking the high type by \$1. The right-hand side is the alternative transfer's inefficiency, $1 - \lambda_H$, scaled by the amount $1/(\Delta\lambda)$ needed to reduce the low type's utility from mimicking the high type by \$1, thus preserving incentive-compatibility.[25]

Proposition 9 (inefficient bonus cap) *Assume (40) and* $q_H/q_L < \Delta\lambda/(1-\lambda_H)$.

1. *Under a bonus cap at any* $\bar{y} \in [y^*, y_H^c]$, *the unique competitive equilibrium is a separating one. Low types receive their symmetric-information contract* $(y^*, z^* = w(y^*) + (B - y^*)\theta_L)$ *while high types get bonus* $\bar{y}$, *a non-monetary transfer* $\zeta_H = [(B - \bar{y})\Delta\theta + w(y^*) - w(\bar{y})]/(1 - \lambda_L)$ *and a monetary transfer* $z_H^r = w(\bar{y}) + (B - \bar{y})\,\theta_H - \zeta_H$.

2. *Social welfare is strictly increasing (in the Pareto sense) with the cap level* $\bar{y}$, *and thus maximized when no binding regulation is imposed* ($\bar{y} = y_H^c$).

The condition on q_H/q_L ensures that the least-cost separating allocation described in the first part of the proposition is interim efficient, and therefore the (unique) equilibrium. The inequality is less likely to be satisfied if the alternative currency is very inefficient (λ_H small), required in large quantities ($\Delta\lambda$ small) to achieve separation, and if the high types are numerous (q_H large). The second result then follows from (40): even at y_H^c, where the marginal bonus distortion is maximal, it is still smaller than that from using the alternative currency. A fortiori, the further down $\bar{y}$ forces y^c, the less is gained in productive efficiency, while the marginal distortion associated with the alternative screening device remains constant.[26]

When the regulator is unable to distinguish the performance-related and fixed parts of compensation, the only cap he can impose is on total earnings Y. Such regulations are also counterproductive when firms have relatively easy access (e.g., at constant marginal cost) to alternative rewards that allow them to screen and compensate high types.

Proposition 10 (inefficient compensation cap) *Let total earnings be capped at any level* $\bar{Y}$ *low enough to be binding on the high type's compensation in an unregulated equilibrium, but not on what they would earn in a first-best situation where types are observable:*[27]

$$Aa(y^*) + Bb(y^*) + B\theta_H \leq \overline{Y} \leq Aa(y_H^c) + Bb(y_H^c) + B\theta_H = Y_H^c \tag{41}$$

[25] Formally, consider the utility $U_H = w(y_H) + B\theta_H - (1-\lambda_H)\zeta_H$ offered to high types by a zero-profit-making firm that pays them ζ_H in the alternative currency. Maximizing U_H subject to the low type's incentive constraint $U_L^* \geq U_H - y_H\Delta\theta - \zeta_H\Delta\lambda$ leads, under (40), to $y_H = y_H^c$ and $\zeta_H = 0$.

[26] This strong assumption is the polar opposite of that in Proposition 8 ($\zeta_H \equiv 0$). More generally, ζ could have increasing marginal cost, rising toward infinity at some finite $\bar{\zeta} > 0$.

[27] These inequalities means that the cap binds on high-skill workers' compensation Y_H^c in an unregulated equilibrium, but not on what they would be paid in a first-best situation where types are observable. The latter also implies, by Part 2 above, that the cap is not binding on low-skill workers' compensation Y_L^c in an unregulated equilibrium.

1. *If q_L is high enough, the unique equilibrium is the LCS allocation in which low types receive their symmetric-information contract $(y^*, z^* = w(y^*) + (B - y^*)\theta_L)$ while high types get a "package $(y_H^r, \zeta_H^r, z_H^r)$ given by*

$$\begin{aligned} w(y^*) - w(y_H^r) &= (B - y_H^r)\Delta\theta - (1 - \lambda_L)\left[Aa(y_H^r) + Bb(y_H^r) + B\theta_H - \overline{Y}\right], \\ \zeta_H &= \pi(y_H^r) + B\theta_H + y_H^r b(y_H^r) - \overline{Y}, \\ z_H^r &= \bar{Y} - [\theta_H + b(y_H^r)]\, y_H^r. \end{aligned}$$

2. *Any tightening of the earnings cap (reduction in $\overline{Y}$) leads to a Pareto deterioration.*

Although a confiscatory tax of 100% above a ceiling $\bar{Y}$ is unambiguously welfare reducing, *some* positive amount of taxation is always optimal to improve on the laissez-faire "bonus culture". While characterizing the optimal tax in this setting is complicated and left for future work, we can show:

Proposition 11 *A small tax τ on total earnings always improves welfare: $dW/d\tau|_{\tau=0} > 0$.*

The intuition is as follows. To start with, condition (40) ensures that, for τ sufficiently small, the firm does not find it profitable to resort to inefficient transfers, hence still uses performance pay to screen workers. Taxing total earnings then has two effects. First, under symmetric information, it distorts (net) incentives downward relative to the private and social optimum, y^*. Second, it shrinks the compensation differential received by the two types under any given contract. This reduces low types' incentive to mimic high ones, thus dampening firms' need to screen through high-powered (net) incentives and thereby alleviating the misallocation of effort. For small τ the first effect is of second-order (a standard Harberger triangle), whereas the second one is of first order, due again to the externality between firms discussed earlier.

4.2 Multidimensional incentives and noisy performance measurement

Performance in activity A was so far taken to be non-measurable or non-contractible. Consequently, effort a was driven solely by intrinsic motivation, or by fixed outside incentives such as potential legal liability or reputational concerns. In the other version of the multitask problem studied by Holmström and Milgrom (1991), every dimension of performance can be measured but with noise, and this uncertainty limits the extent to which risk-averse agents can be incentivized. We now extend our theory to this case, where there need not be any intrinsic motivation. This variant of the model is particularly applicable to the issue of short- versus long-term performance and the possible recourse to *deferred compensation,* clawbacks and other forms of *long-term pay,*

Outputs in tasks A and B are now $\theta^A + a + \varepsilon^A$ and $\theta^B + b + \varepsilon^B$, where θ^A, θ^B are the employee's talents in each task, a and b his efforts as before, and ε^A, ε^B independent random shocks with $\varepsilon^A \sim \mathcal{N}(0, \sigma_A^2)$ and $\varepsilon^B \sim \mathcal{N}(0, \sigma_B^2)$. A compensation package is a triple (y^A, y^B, z) where y^A and y^B are the bonuses on each task and z the fixed wage. As in Holmström and Milgrom (1991), agents have mean-variance preferences. Letting r denote the index of risk aversion, utility is thus:

$$U(a,b;\theta^A,\theta^B,y,z) = (\theta^A + a)y^A + (\theta^B + b)y^B + z - C(a,b) - \frac{r}{2}\left[(y^A)^2\sigma_A^2 + (y^B)^2\sigma_B^2\right], \qquad (42)$$

with the cost function having the same properties as before. Given an incentive vector $y \equiv (y^A, y^B)$, the agent chooses efforts $a(y)$ and $b(y)$ that jointly solve $C_a(a(y), b(y)) = y^A, C_b(a(y), b(y)) = y^B$; it is easily verified that $a(y)$ is increasing in y^A and decreasing in y^B, while $b(y)$ has the opposite properties. The firm's profit function remains unchanged, so total surplus is $w(y) + A\theta^A + B\theta^B$, where the allocative component is now equal to

$$w(y) \equiv Aa(y) + Bb(y) - C(a(y), b(y)) - \frac{r}{2}\left[(y^A)^2\sigma_A^2 + (y^B)^2\sigma_B^2\right]. \qquad (43)$$

Assuming strict concavity and an interior solution, the vector of first-best bonuses $y^* \equiv (y^{A*}, y^{B*})$ solves the first-order conditions:

$$\frac{\partial w}{\partial y^A}\left(y^{A*}, y^{B*}\right) = \frac{\partial w}{\partial y^B}\left(y^{A*}, y^{B*}\right) = 0, \qquad (44)$$

which is shown in the appendix to imply that $y^{A*} < A$ and $y^{B*} < B$.

There are again two types of workers, H and L, in proportions q_H and q_L, who each select their preferred contract from the menus $\left\{(y_i^A, y_i^B, z_i)\right\}_{i=H,L}$ offered by firms. Denoting $\Delta y^\tau \equiv y_H^\tau - y_L^\tau$ and $\Delta\theta^\tau \equiv \theta_H^\tau - \theta_L^\tau$ for each task $\tau = A, B$, incentive compatibility requires that

$$\sum_{\tau=A,B}(\Delta y^\tau)(\Delta\theta^\tau) \geq 0. \qquad (45)$$

To simplify the analysis, we assume H types to be more productive in both tasks: $\Delta\theta^A \geq 0$ and $\Delta\theta^B > 0$ (otherwise, which type is "better" depends on the slopes of the incentive scheme).

- *Monopsony.* Denoting $D_i \equiv A\theta_i^A + B\theta_i^B$ for $i = H, L$, a monopsonistic employer solves

$$\begin{aligned}
&\max\{q_H\left[w(y_H) + D_H - U_H\right] + q_L\left[w(y_L) + D_L - U_L\right]\}, \text{ subject to} \\
&U_L \geq \overline{U}, \\
&U_H \geq U_L + y_L^A\Delta\theta^A + y_L^B\Delta\theta^B.
\end{aligned}$$

This yields $y_H^m = y^*$, while y_L^m is given by

$$\frac{1}{\Delta\theta^A}\frac{\partial w}{\partial y^A}\left(y_L^m\right) = \frac{1}{\Delta\theta^B}\frac{\partial w}{\partial y^B}\left(y_L^m\right) = -\frac{q_H}{q_L}. \qquad (46)$$

As before, the incentives of low types (only) are distorted downward, now in both activities. Note also how the efficiency losses, normalized by their offsetting rent reductions, are equalized across the two tasks. As before, one can show that it is indeed optimal to employ both types as long as q_L is above some cutoff $\underline{q}_L < 1$, which we shall assume.

- *Perfect competition.* We look again for a least-cost separating equilibrium. Denoting $U_L^{SI} \equiv$

$w(y^*) + D_L$ type L's symmetric-information utility, such an allocation must solve:

$$\begin{aligned}
&\max\{U_H\}, \text{ subject to} \\
&U_H = w(y_H) + D_H, \\
&U_L^{SI} \geq U_H - y_H^A \Delta\theta^A - y_H^B \Delta\theta^B.
\end{aligned}$$

Let κ^c denote the shadow cost of the second constraint. The first-order conditions are then

$$\frac{1}{\Delta\theta^A} \frac{\partial w(y_H)}{\partial y_H^A} = \frac{1}{\Delta\theta^B} \frac{\partial w(y_H)}{\partial y_H^B} = -\kappa^c, \tag{47}$$

while the binding incentive constraint takes the form

$$w(y^*) - w(y_H) = \left(A - y_H^A\right) \Delta\theta^A + \left(B - y_H^B\right) \Delta\theta^B. \tag{48}$$

Hence, a system of three equations determining $(y_H^{A,c}, y_H^{B,c}, \kappa^c)$, independently of the prior probabilities, as usual for the LCS allocation. Clearly, high-ability agents are again overincentivized, now in both tasks. Note also that even though competitive firms and monopsonist use screening for very different purposes, resulting in opposite types of distortions, both equalize those distortions (properly normalized by unit rents) across the two tasks.

The LCS allocation is, once again, the (unique) equilibrium if and only if it is interim efficient. In the appendix we generalize Lemma 1 to show:

Lemma 2 *The LCS allocation y_H^c is interim efficient if and only if*

$$\frac{1}{\Delta\theta^A} \frac{\partial w(y_H^c)}{\partial y_H^A} = \frac{1}{\Delta\theta^B} \frac{\partial w(y_H^c)}{\partial y_H^B} \geq -\frac{q_L}{q_H} \tag{49}$$

or, equivalently, $\kappa^c \leq q_L/q_H$.

This condition generalizes (23) and has the same interpretation, which can now be given in terms of either task. Intuitively, the larger the distortion in the partial derivatives, the higher the welfare loss relative to first best; condition (49) requires that it not be so large as to render profitable a deviation to a more efficient contract sustained by cross-subsidies.

- *Competition vs. monopsony.* Competition yields lower welfare when $L^m = q_L\left[w(y^*) - w\left(y_L^m\right)\right] < q_H\left[w(y^*) - w(y_H^c)\right] = L^c$. One simple case in which this occurs is, as before, when q_H/q_L is small enough. Indeed $w(y^*) - w(y_H^c)$ is independent of this ratio, whereas under monopsony the distortion becomes small as the high types from whom it seeks to extract rents become more scarce: as q_H/q_L tends to zero, (46) shows that y_L^m tends to y^* and $y^* - y_L^m$ is of order (q_H/q_L). Therefore $w(y^*) - w\left(y_L^m\right)$ is of order $(q_H/q_L)^2$, implying that $L^m << L^c$.

Proposition 12 *There exist q_H^{**} such that for all $q_H \leq q_H^{**}$, welfare is higher under monopsony than under competition.*

• *Quadratic cost.* This specification allows for many further results, particularly on comparative statics. First, effort levels are $a = \left(y^A - \gamma y^B\right)/\left(1-\gamma^2\right)$ and $b(y) = \left(y^B - \gamma y^A\right)/\left(1-\gamma^2\right)$, which are non-negative as long as $y^A/y^B \in [\gamma, 1/\gamma]$. Next, note that the first-order conditions of the first-best, monopsony and competitive problems lead to very similar systems of linear equations,

$$A - \gamma B - y_H^A + \gamma y_H^B - r\left(1-\gamma^2\right)\sigma_A^2 y_H^A = -\tilde{\kappa}\left(1-\gamma^2\right)\Delta\theta^A, \quad (50)$$

$$B - \gamma A - y_H^B + \gamma y_H^A - r\left(1-\gamma^2\right)\sigma_B^2 y_H^B = -\tilde{\kappa}\left(1-\gamma^2\right)\Delta\theta^B, \quad (51)$$

with the only difference being that $\tilde{\kappa} \equiv 0$ in the first case, $\tilde{\kappa} \equiv q_H/q_L$ in the second, and $\tilde{\kappa} = \kappa^c$ in the third. In particular, the first-best solution is

$$y^{A*} = \frac{r\sigma_B^2\left(A - \gamma B\right) + A}{1 + r\left(\sigma_A^2 + \sigma_B^2\right) + (1-\gamma^2)r^2\sigma_A^2\sigma_B^2} < A, \quad (52)$$

and a similar formula for $y^{B*} < B$, obtained by permuting the roles of A and B. The condition $\gamma \leq y_*^A/y_*^B \leq 1/\gamma$, which ensures that $a(y^*) \geq 0$ and $b(y^{*)} \geq 0$, is then equivalent to[28]

$$\frac{\gamma r\sigma_A^2}{1 + r\sigma_B^2} \leq \frac{A - \gamma B}{B - \gamma A} \leq \frac{1 + r\sigma_A^2}{\gamma r\sigma_B^2}. \quad (53)$$

The properties of this first-best benchmark parallel those in Holmström and Milgrom (1991).

Proposition 13 *The first-best incentive y^{A*} is decreasing in B in σ_A^2, and conversely increasing in A and σ_B^2, whereas y^{B*} has the opposite properties. Both are decreasing in risk aversion, r.*

Turning next to monopsony and competition, the system (50)-(51) can also be rewritten in terms of the price distortions $y^\tau - y^{\tau*}$, $\tau = A, B$, leading to the following set of results.

Proposition 14 (incentive distortions) *The relative overincentivization of task B compared to task A induced by competition is equal to the relative underincentivization of task B compared to task A induced by monopsony:*

$$\frac{y_H^{B,c} - y^{B*}}{y_H^{A,c} - y^{A*}} = \frac{y^{B*} - y_L^{B,m}}{y^{A*} - y_L^{A,m}} = \frac{\left[1 + r\left(1-\gamma^2\right)\sigma_A^2\right]\Delta\theta^B + \gamma\Delta\theta^A}{\left[1 + r\left(1-\gamma^2\right)\sigma_B^2\right]\Delta\theta^A + \gamma\Delta\theta^B} \equiv \rho\left(\sigma_A^2, \sigma_B^2; \Delta\theta^A/\Delta\theta^B; r\right). \quad (54)$$

It is greater:

(i) The greater the noise σ_A^2 in task A and the the lower the noise σ_B^2 in task B;

(ii) The greater the comparative advantage $\Delta\theta^B/\Delta\theta^A$ of H types in task B, relative to task A;

(iii) The greater workers' risk aversion if $\sigma_A^2/\sigma_B^2 > \Delta\theta^A/\Delta\theta^B$ (and the smaller if not).

[28] An alternative way of ensuring that a remains non-negative (allowing σ_A^2 to become arbitrary large) is of course to incorporate intrinsic motivation $v_a a$ into (42), with $v_a \geq \gamma B$. The model then nests that of Section 2 as a limiting case for $(\sigma_A^2, \sigma_B^2) \to (+\infty, 0)$. Alternatively, $a < 0$ (say) may be interpreted as nefarious or antisocial activities (stealing coworkers' ideas, devising schemes to deceive customers, et.) that require effort but allow the agent to increase his performance –and bonus earned– in the B dimension.

These results are intuitive: more *noisy measurement* makes a task a *less efficient screening device* –whether for rent-extraction or employee-selection purposes– while a higher *ability differential* of low and high types makes it a more efficient one. As to the "mirror image" property of relative price wedges under monopsony and competition, it reflects the fact that both types of firms equalize the (normalized) marginal distortions across the two tasks.[29]

We next consider workers' effort allocations.

Proposition 15 (effort distortions) *(1) Competition distorts high-skill agents' effort ratio away from task A, and monopsony away from task B,* $a(y_H^c)/b(y_H^c) < a(y^*)/b(y^*) < a(y_L^m)/b(y_L^m)$*, if and only if*

$$\frac{A - \gamma B}{B - \gamma A} > \frac{\Delta\theta^A}{\Delta\theta^B}. \tag{55}$$

(2) Competition reduces the absolute level of effort on task A, $a(y_H^c) < a(y^*)$*, while increasing that on task B (and monopsony has the opposite effects), if and only if*

$$\frac{\gamma r \sigma_A^2}{1 + r\sigma_B^2} > \frac{\Delta\theta^A}{\Delta\theta^B}. \tag{56}$$

The broad message of these results accords with that of Sections 3.1 and 3.2, but Proposition 15 also yields several new insights about how the misallocation of efforts is shaped by the *measurement error* in each the two tasks, their *substitutability* in effort, high-skill agents' *comparative advantage* in one or the other of them, and the degree of *risk aversion*. The second result is particularly noteworthy: even though *both* tasks are more strongly incentivized under competition, effort in task A declines, because task B becomes disproportionately rewarded.[30]

Finally, we can also state precisely when competition or monopsony is more efficient. As before, we denote $\underline{q}_L$ the threshold value of q_L above which the monopsonist employs both types, $\tilde{q}_L = \kappa^c/(1+\kappa^c)$ the minimum value ensuring that the LCS allocation is the competitive outcome, and $q_L^* \equiv \max\{\underline{q}_L, \tilde{q}_L\}$.

Proposition 16 (social welfare) *Let* $q_L \geq q_L^*$*. Social welfare is then higher under monopsony than under competition if and only if* $q_H/q_L < (\kappa^c)^2$*.*

The condition ensures that $q_L(y^{A*} - y_L^{A,m})^2 < q_H(y_H^{A,m} - y^{A*})^2$, meaning that the losses from the mispricing of activity A are higher under competition. Because the mispricing of B is proportional to that of A, with the same coefficient ρ under monopsony and competition, total losses are also higher.

[29] One can also solve explicitly the system with $\tilde{\kappa} = q_H/q_L$ for the monopsony solution $y_L^m = (y_L^{A,m}, y_L^{B,m})$. In the competitive case, the LCS condition (48) is quadratic in $(y_L^{A,c}, y_L^{B,c})$, so by (54) it reduces to a quadratic equation in $y_L^{A,c}$ only (or equivalently, in κ^c).

[30] Note also that when (53) holds, so that $a(y^*) \geq 0, b(y^*) \geq 0$, (56) implies (55).

4.3 Competition for the motivated

We now return to the benchmark specification of Section 3 (task A is non-contractible, task B is perfectly measurable, agents are risk-neutral) and study the polar case where all workers have the same productivity θ (normalized to 0 without loss of generality) in task B but differ in their "ethical" motivations v for task A : a fraction p_L has $v = v_L$ and the remaining p_H have $v = v_H$.

When facing compensation scheme (y, z), an agent of type v_i has net utility $u_i(y) + z$, where

$$u_i(y) \equiv \max_{(a,b)} \{v_i a + yb - C(a, b)\}. \tag{57}$$

Let $a_i(y)$ and $b_i(y)$ denote the corresponding efforts, namely the solutions to the system $\{C_a(a, b) = v_i$ and $C_b(a, b) = y\}$, and note that $u_i'(y) = b$. Concavity of the cost function implies here again that $a_i'(y) < 0 < b_i'(y)$, as well as $a_L(y) < a_H(y)$ and $b_H(y) < b_L(y)$ in response to any given incentive rate $y > 0$.The employer of an agent with type v_i makes net profit $\pi_i(y) - z$, where

$$\pi_i(y) \equiv A a_i(y) + (B - y) b_i(y). \tag{58}$$

Finally, we denote

$$w_i(y) \equiv u_i(y) + \pi_i(y), \tag{59}$$

$$y_i^* = \arg\max\{w_i(y)\}. \tag{60}$$

In contrast to the case of heterogeneity in talent θ, there are now generally different optimal incentive rates for each type of worker.[31] Next, we note that when confronted with an incentive-compatible menu of options, the more pro-social type (v_H) chooses a less powerful incentive scheme:[32]

$$y_H \leq y_L \quad \text{(and so } z_H \geq z_L).$$

This, in turn, implies that if a_L and a_H denote the two types' equilibrium efforts on task A, then $a_L \leq a_H$. The more pro-socially inclined employee exerts more effort on task A both because he is more motivated for it and because he chooses a lower-powered incentive scheme.

• *Monopsony.* The monopsonist offers an incentive-compatible menu (y_L, z_L) and (y_H, z_H), or equivalently (y_L, U_L) and (y_H, U_H) so as to solve:

$$\begin{aligned}
&\max_{\{(y_i,\ z_i)\}_{i=H,L}} \Big\{ \sum_{i=H,L} p_i \left[w_i(y_i) - U_i\right] \Big\}, \text{ subject to} \\
&U_L \geq \overline{U} \\
&U_H \geq U_L + u_H(y_L) - u_L(y_L) \\
&U_L \geq U_H + u_L(y_H) - u_H(y_H).
\end{aligned}$$

[31] In the quadratic-cost benchmark, $y_L^* = y_H^* = B - \gamma A$. In general, the variation of y with v involves the third derivatives of C and is thus ambiguous.

[32] Adding up the two incentive constraints, $U_H \geq U_L + u_H(y_L) - u_L(y_L)$ and $U_L \geq U_H - u_H(y_H) + u_L(y_H)$, yields $0 \geq \int_{y_H}^{y_L} [u_H'(y) - u_L'(y)]\, dy = \int_{y_H}^{y_L} [b_H(y) - b_L(y)]\, dy$. Since $b_H(y) < b_L(y)$ for all y, the result follows.

The first two constraints must clearly be binding, while the third imposes $y_H \leq y_L$, as seen above. Substituting in, the solution satisfies

$$y_H^m = y_H^* \tag{61}$$

$$w_L'(y_L) = \frac{p_H}{p_L}\left[u_H'(y_L) - u_L'(y_L)\right] = \frac{p_H}{p_L}\left[b_H(y) - b_L(y)\right] \tag{62}$$

when it is interior; more generally, the left-hand side of (62) must be no greater than the right-hand side. Since the latter is strictly negative, one must have in any case

$$y_L^m > y_L^*. \tag{63}$$

The monopsonist offers a higher-powered incentive scheme than under symmetric information so as to limit the rent of the more prosocial types, who clearly benefits less from an increase in y.

• *Perfect competition.* Because employees' intrinsic-motivation benefits va are private, firms have no reason to compete to select more prosocial types. As a result, the kind of incentive distortion seen earlier does not arise, and the competitive equilibrium is the symmetric-information outcome. Employers offer the menu $\{(y_i^*,\ z_i^*)\}_{i=H,L}$, where for each type y_i^* is the efficient incentive rate defined by (60) and $z_i^* \equiv \pi_i(y_i^*)$, leaving the firm with zero profit. Type $i = H, L$ then chooses

$$\max_{j\in\{H,L\}} \left\{u_i(y_j^*) + \pi_i(y_j^*) = w_i(y_j^*)\right\}. \tag{64}$$

By definition, $j = i$ is the optimal choice, so the symmetric-information outcome is indeed incentive compatible.

Proposition 17 *When agents are similar in measurable talent θ but differ in their ethical values v, monopsony leads to an overincentivization of low-motivation types, $y_L^m > y_L^*$ (with $y_H^m = y_H^*$), whereas competition leads to the first-best outcome, $y_L^c = y_L^*$, $y_H^c = y_H^*$.*

Would conclusions differ under an alternative specification of the impact of prosocial heterogeneity? Suppose that instead of enjoying task A more, a more prosocial agent supplies more unmeasured positive externalities on the firm (or on her coworkers, so that their productivity is higher, or their wages can be reduced due to a better work environment). In other words, agents $i = H, L$ share the same preferences,

$$u_i(y) = \max_{(a,b)}\{\bar{v}a + yb - C(a,b)\} \equiv u(y)$$

but have different productivities in the A activity,

$$\pi_i(y) = A\left(a(y) + \nu_i\right) + (B - y)b(y). \tag{65}$$

Under this formulation there is no way to screen an agent's type, so the outcome under both monopsony and competition is full pooling at the efficient incentive power:

$$y_i^* = y^* \equiv \arg\max\{Aa(y) + Bb(y) - C\left(a(y), b(y)\right)\}. \tag{66}$$

Sorting will occur, on the other hand, when agents' intrinsic motivation is not unconditional, as we have assumed, but reciprocal –that is, dependent on the presence in the same firm of other people who act cooperatively (e.g., Kosfeld and von Siemens (2011)), or on the firm fulfilling a socially valuable mission rather than merely maximizing profits (e.g., Besley and Ghatak 2005, 2006, Brekke and Nyborg 2006). The fact that the benefits of "competing for the motivated" are somewhat attenuated in our model with respect to those only reinforces the contrast with the potentially very distortionary effects of competition for "talent", thus further strengthening our main message.

5 Conclusion

This paper has examined how the extent of labor market competition affects the structure of incentives, multitask efforts and outcomes such as short- and long-run profits, earnings inequality and aggregate efficiency. The analysis could be fruitfully extended in several directions.

First, one could analyze increased competition as a reduction in fixed costs and examine whether there is too little or too much entry into the market. The modeling device of co-located outside options we introduced into the linear Hotelling model should work for the circular one as well. More generally, it could prove useful in other settings, as it allows for a clean separation between intra- and inter-market (or brand) competition and ensures that the market remains covered at all levels of competitiveness between Bertrand and monopoly.

A second extension is to allow for asymmetries between firms or sectors. For instance, task unobservability may be less of a concern for some (e.g., private-equity partnerships) and more for others (large banks), but if they compete for talent the high-powered incentives efficiently offered in the former may spread to the latter, and do damage there. Heterogeneity also raises the question of the self-selection of agents into professions and their matching with firms or sectors, e.g., between finance and science or engineering.

Our analysis has focused on increased competition in the labor market, but similar effects could arise from changes in the product market. One can thus envision settings in which high-skill workers become more valuable as firms compete harder for customers, for instance because the latter become more sensitive to quality. Finally, the upward pressure exerted on pay by competition could also result in agents motivated primarily by monetary gain displacing intrinsically motivated ones within (some) firms, potentially resulting in a different but equally detrimental form of "bonus culture". This idea is pursued in Bénabou and Tirole (2013).

Appendix A: Quadratic-Cost Case

Let the cost function be

$$C(a,b) = a^2/2 + b^2/2 + \gamma ab. \tag{A.1}$$

with $\gamma^2 < 1$, ensuring convexity. The main case of interest is $\gamma > 0$ (efforts are substitutes), but all derivations and formulas hold with $\gamma < 0$ (complements) as well. The first-order conditions for (2) yield $v = a + \gamma b$, and $y = b + \gamma a$, hence

$$a(y) = \frac{v - \gamma y}{1-\gamma^2},\ b(y) = \frac{y - \gamma v}{1-\gamma^2},\ \text{and } a'(y) = \frac{-\gamma}{1-\gamma^2},\quad b'(y) = \frac{1}{1-\gamma^2}. \tag{A.2}$$

Equations (10)-(11) then lead to

$$y^* = B - \gamma A, \tag{A.3}$$

$$w(y^*) - w(y) = -\int_{y^*}^{y} w'(z)\, dz = -\int_{y^*}^{y} \left(\frac{y^* - z}{1-\gamma^2}\right) dz = \frac{(y-y^*)^2}{2\,(1-\gamma^2)}. \tag{A.4}$$

1. Monopsony. Substituting the last two expressions into Proposition 1 yields

$$y_L^m = y^* - (1-\gamma^2)\frac{q_H}{q_L}\Delta\theta, \tag{A.5}$$

$$L^m = \frac{1}{2}\frac{q_H^2}{q_L}(1-\gamma^2)(\Delta\theta)^2. \tag{A.6}$$

2. Perfect competition. From (A.4) and (22), we get:

$$\frac{1}{2\,(1-\gamma^2)}(y_H^c - y^*)^2 = (B - y_H^c)\Delta\theta. \tag{A.7}$$

Let $\nu \equiv y_H^c - y^* = y_H^c - B + \gamma A > 0$ and $\omega \equiv \left(1-\gamma^2\right)\Delta\theta$. Then $\nu^2 + 2\omega\,(\nu - \gamma A) = 0$ and solving this polynomial yields $\nu = -\omega + \sqrt{\omega^2 + 2\omega\gamma A} > 0$, or

$$y_H^c = B - \gamma A - \omega + \sqrt{\omega^2 + 2\omega\gamma A}. \tag{A.8}$$

Note that $y_H^c < B$, since $\omega + \gamma A > \sqrt{\omega^2 + 2\omega\gamma A}$. The resulting efficiency loss relative to the social optimum is

$$L^c = q_H\,[w(y^*) - w(y_H^c)] = (B - y_H^c)\,q_H\Delta\theta = \left(\gamma A + \omega - \sqrt{\omega^2 + 2\omega\gamma A}\right) q_H\Delta\theta. \tag{A.9}$$

Finally, the least-cost separating allocation is interim efficient if (23) holds, which here becomes

$$\frac{1}{1-q_L} \geq \sqrt{1 + \frac{2\gamma A}{\omega}}. \tag{A.10}$$

It will be useful to rewrite the condition as

$$\frac{\gamma A}{\omega} < \frac{1}{2}\left(\frac{q_L}{q_H}\right)^2 + \frac{q_L}{q_H}. \tag{A.11}$$

3. Welfare under monopsony versus competition. Using (A.6) and (A.9), condition (26) becomes:

$$\left(\frac{q_H^2}{2q_L}\right)\left(1-\gamma^2\right)(\Delta\theta)^2 < (B - y_H^c)q_H\Delta\theta \iff \frac{q_H}{2q_L}\omega < \gamma A - \nu \iff \nu < \gamma A - \frac{q_H}{2q_L}\omega.$$

Substituting into the polynomial whose positive root is ν, this is equivalent to:

$$\left(\gamma A - \frac{q_H}{2q_L}\omega\right)^2 > 2\omega\left(\frac{q_H}{2q_L}\omega\right) = \frac{q_H}{q_L}\omega^2,$$

which yields (27). This inequality and the interim efficiency condition (A.11) are simultaneously satisfied if and only if

$$\underline{M}(q_L) \equiv \frac{1-q_L}{2q_L} + \sqrt{\frac{1-q_L}{q_L}} < \frac{\gamma}{1-\gamma^2}\frac{A}{\Delta\theta} \leq \frac{1}{2}\left(\frac{q_L}{1-q_L}\right)^2 + \frac{q_L}{1-q_L} \equiv \bar{M}(q_L). \tag{A.12}$$

Note that:

(i) $\underline{M}(q_L) < \bar{M}(q_L)$ if and only if $q_H/q_L < 1$, so for any $q_L > 1/2$, (A.12) defines a nonempty range for $(A/\Delta\theta)\left[\gamma/(1-\gamma^2)\right]$.

(ii) As $q_L \to 1$, $\underline{M}(q_L) \to 0$ and $\bar{M}(q_L) \to +\infty$, so arbitrary values of $(A/\Delta\theta)\left[\gamma/(1-\gamma^2)\right]$ become feasible, including arbitrarily large values of A or arbitrarily low values of $\Delta\theta$. In particular, imposing $\gamma A < B(1-\gamma^2) - q_H\Delta\theta/q_L$ to ensure $0 < y^* < y_L^m$ is never a problem for q_L large enough.

4. Imperfect competition. In Region I, $\hat{y}_H(t)$ is defined as the solution to (C.22) in Appendix C, which here becomes:

$$\begin{aligned}\frac{(y_H - y^*)^2}{2(1-\gamma^2)} - (B - y_H)\Delta\theta + \frac{t}{q_L\Delta\theta}\frac{(y_H - y^*)}{(1-\gamma^2)} &= 0 \iff \\ \nu^2 + 2\left(t/q_L\Delta\theta\right)\nu - 2\omega(\gamma A - \nu) &= \nu^2 + 2(\omega + t/q_L\Delta\theta)\nu - 2\omega\gamma A = 0,\end{aligned}$$

with the above definitions of ω and $\nu = y_H - y^*$. Solving, we have:

$$\hat{y}_H(t) = B - \gamma A - t/q_L\Delta\theta - \omega + \sqrt{(\omega + t/q_L\Delta\theta)^2 + 2\omega\gamma A}.$$

It is easily verified that $\hat{y}_H(0) = y_H^c$ and

$$q_L\Delta\theta \cdot \hat{y}_H'(t) = -1 + \frac{\omega + t/q_L\Delta\theta}{\sqrt{(\omega + t/q_L\Delta\theta)^2 + 2\omega\gamma A}} < 0. \tag{A.13}$$

Moreover, this expression is increasing in t/q_L, so $\hat{y}_H(t)$ is decreasing and convex over Region I.

In Region II, $\hat{y}_H(t)$ is defined as the solution to (C.25) in Appendix 5, which here becomes:

$$\begin{aligned}
w(y^*) + \theta_L B - \frac{(y_H - y^*)^2}{2(1-\gamma^2)} + (B - y_H)\Delta\theta - \frac{t}{\Delta\theta}\frac{(y_H - y^*)}{(1-\gamma^2)} - \bar{U} - t &= 0 \iff \\
\frac{\nu^2}{2(1-\gamma^2)} + (\nu - \gamma A)\Delta\theta + \frac{t}{\Delta\theta}\frac{\nu}{(1-\gamma^2)} + \theta_L B + \bar{U} + t - w(y^*) &= 0 \iff \\
\nu^2 + 2(\nu - \gamma A)\omega + \frac{2t\nu}{\Delta\theta} + 2(1-\gamma^2)\left(\bar{U} + t + \theta_L B - w(y^*)\right) &= 0 \iff \\
\nu^2 + 2\nu(\omega + t/\Delta\theta) - 2\gamma A\omega + 2(1-\gamma^2)\left(\bar{U} + t + \theta_L B - w(y^*)\right) &= 0.
\end{aligned}$$

Solving, we have:

$$\hat{y}_H(t) = B - \gamma A - t/\Delta\theta - \omega + \sqrt{(\omega + t/\Delta\theta)^2 + 2[\omega\gamma A + (1-\gamma^2)\left(\bar{U} + t + \theta_L B - w(y^*)\right)]}. \quad \text{(A.14)}$$

Moreover,

$$\hat{y}'_H(t) = -1 + \frac{\omega + t/\Delta\theta}{\sqrt{(\omega + t/\Delta\theta)^2 + 2[\omega\gamma A + (1-\gamma^2)\left(\bar{U} + t + \theta_L B - w(y^*)\right)]}} < 0$$

and it is increasing in $t/\Delta\theta$, so $\hat{y}_H(t)$ is decreasing and convex over Region II.

In Region III, $\hat{y}_L(t)$ is defined as the solution to (C.29). Denoting $\nu = y_L - y^*$, this now becomes:

$$\begin{aligned}
w(y^*) + \theta_L B - \bar{U} - t + (\gamma A - \nu)\Delta\theta - \frac{tq_L}{q_H\Delta\theta}\frac{\nu}{(1-\gamma^2)} &= 0 \iff \\
(1-\gamma^2)\left[w(y^*) + \theta_L B - \bar{U} - t + \gamma A\Delta\theta\right] &= \nu\left(\omega + \frac{tq_L}{q_H\Delta\theta}\right).
\end{aligned}$$

Solving, we have:

$$\hat{y}_L(t) = B - \gamma A - \frac{(1-\gamma^2)\left[\bar{U} + t - w(y^*) - \theta_L B\right] - \omega\gamma A}{\omega + tq_L/q_H\Delta\theta}. \quad \text{(A.15)}$$

It is easily verified that $\hat{y}_L(+\infty) = B - \gamma A - (1-\gamma^2)q_H\Delta\theta/q_L = y_L^m$. Moreover, recalling (12),

$$\hat{y}'_H(t) = \frac{(q_L/q_H\Delta\theta)\left[\bar{U} - w(y^*) - \theta_L B - \omega\gamma A\Delta\theta\right] - \omega}{\left(\omega + tq_L/q_H\Delta\theta\right)^2/(1-\gamma^2)} < 0$$

and this function is increasing in $t/\Delta\theta$, implying that $\hat{y}_L(t)$ is convex. ■

Welfare effects of transport costs. Let $W(t) = w(y_H(t)) + q_L w(y_L(t)) + B\bar{\theta}$ and $\tilde{W}(t) \equiv W(t) - t/4$. By Proposition 5, $W'(t) > 0$ for all t. We now find conditions ensuring that $\tilde{W}'(t) > 0$ for t small enough. For $t \leq t_1, W'(t) = q_H w'(\hat{y}_H(t))\hat{y}'_H(t)$. With quadratic costs, and using (A.4), (A.8) and (A.13), $q_H W'(0)\hat{y}'_H(0) = q_H\left(\sqrt{1 + 2\gamma A/\omega} - 1\right)\left(1 - 1/\sqrt{1 + 2\gamma A/\omega}\right)$, which for small $\Delta\theta$ is equivalent to $q_H\sqrt{2\gamma A/\omega}$. As seen from (A.10), interim efficiency requires $q_H \leq (1 + 2\gamma A/\omega)^{-1/2} \approx \sqrt{\omega/2\gamma A}$. Letting $q_H \lessapprox \sqrt{\omega/2\gamma A}$ yields $q_H W'(0)\hat{y}'_H(0) \approx 1 > 1/4$, hence the result. ■

Proof of Proposition 14. Subtracting the first-best solution from (50)-(51) yields

$$
\begin{aligned}
-\left[1+^2\left(1-\gamma^2\right)\sigma_A^2\right]x^A+\gamma x^B &= -\tilde{\kappa}\left(1-\gamma^2\right)\Delta\theta^A, \quad (A.16)\\
\gamma x^A-\left[1+^2\left(1-\gamma^2\right)\sigma_B^2\right]x^B &= -\tilde{\kappa}\left(1-\gamma^2\right)\Delta\theta^B, \quad (A.17)
\end{aligned}
$$

from which ρ is easily obtained. Its comparative statics follow from direct computation. ■

Proof of Proposition 15. (1) It easily seen that $a(y_H)/b(y_H) < a(y^*)/b(y^*)$ if and only if $y_*^B/y_*^A < x_H^B/x_H^A$. Using (52) and (54), this means:

$$\frac{\left[1+r\left(1-\gamma^2\right)\sigma_B^2\right]\Delta\theta^A+\gamma\Delta\theta^B}{\left[1+r\left(1-\gamma^2\right)\sigma_A^2\right]\Delta\theta^B+\gamma\Delta\theta^A} < \frac{r\sigma_B^2\left(A-\gamma B\right)+A}{r\sigma_A^2\left(B-\gamma A\right)+B}.$$

This can be rewritten as

$$
\begin{aligned}
\frac{\Delta\theta^A}{\Delta\theta^B} &< \frac{\left[1+r\left(1-\gamma^2\right)\sigma_A^2\right]\left[r\sigma_B^2\left(A-\gamma B\right)+A\right]-\gamma\left[r\sigma_A^2\left(B-\gamma A\right)+B\right]}{\left[1+r\left(1-\gamma^2\right)\sigma_B^2\right]\left[r\sigma_A^2\left(B-\gamma A\right)+B\right]-\left[\gamma r\sigma_B^2\left(A-\gamma B\right)+A\right]}\\
&= \frac{\left[1+r\left(1-\gamma^2\right)\sigma_A^2\right]r\sigma_B^2\left(A-\gamma B\right)+A-\gamma B+r\sigma_A^2\left[A-\gamma B\right]}{\left[1+r\left(1-\gamma^2\right)\sigma_B^2\right]r\sigma_A^2\left(B-\gamma A\right)+B-\gamma A+r\sigma_B^2\left(B-\gamma A\right)},
\end{aligned}
$$

which simplifies to (55).

(2) We have $a(y_H^c) < a(y^*)$ if and only if $x_H^A < \gamma x_H^B$, that is, $\gamma\rho > 1$, which occurs when

$$
\begin{aligned}
\gamma\left[1+r\left(1-\gamma^2\right)\sigma_A^2\right]\Delta\theta^B+\gamma^2\Delta\theta^A &> \left[1+r\left(1-\gamma^2\right)\sigma_B^2\right]\Delta\theta^A+\gamma\Delta\theta^B \iff\\
r\left(1-\gamma^2\right)\left[\gamma\sigma_A^2\Delta\theta^B-\sigma_B^2\Delta\theta^A\right] &> \left(1-\gamma\right)^2\Delta\theta^A \iff r\left[\gamma\sigma_A^2\Delta\theta^B-\sigma_B^2\Delta\theta^A\right] > \Delta\theta^A,
\end{aligned}
$$

which yields (56). Furthermore, $b(y_H^c) > b(y^*)$ if only if $x_H^B > \gamma x_H^A$, i.e. $\rho > \gamma$, which is implied by $\gamma\rho > 1$. Note, on the other hand, that competition always increases total gross output above the efficient level, $Aa(y^*)+Bb(y^*) < Aa(y_H^c)+Bb(y_H^c)$, if only if $0 < A\left(x_H^A-\gamma x_H^B\right)+B\left(x_H^B-\gamma x_H^A\right)$, or equivalently since $x_H^A > 0: 0 < A\left(1-\rho\gamma\right)+B\left(\rho-\gamma\right) = A-\gamma B+\rho(B-\gamma A)$,which always holds. For a monopsonist $x_L^A < 0$, so the same condition yields $Aa(y_L^m)+Bb(y_L^m) < Aa(y^*)+Bb(y^*)$.■

Proof of Proposition 16. Since $w(y)$ is quadratic and minimized at y^*,

$$
\begin{aligned}
w(y^*)-w\left(y_L^m\right) &= \frac{1}{2}\left(y^*-y_L^m\right)^T\cdot H(w)|_{y^*}\cdot\left(y^*-y_L^m\right) = \frac{\left(x_m^A\right)^2}{2}\left(1\quad\rho\right)\ H(w)|_{y^*}\ ^T(1\quad\rho),\\
w(y^*)-w\left(y_H^c\right) &= \frac{1}{2}\left(y^*-y_H^c\right)^T\cdot H(w)|_{y^*}\cdot\left(y^*-y_H^c\right) = \frac{\left(x_c^A\right)^2}{2}\left(1\quad\rho\right)\ H(w)|_{y^*}\ ^T(1\quad\rho),
\end{aligned}
$$

where we used Proposition (15). Therefore, $q_L\left[w(y^*)-w\left(y_L^m\right)\right] < q_H\left[w(y^*)-w\left(y_H^c\right)\right]$ if and only if $q_L\left(x_m^A\right)^2 < q_H\left(x_c^A\right)^2$. We saw earlier that $x_m^A/x_c^A = x_m^B/x_c^B = (q_H/q_L\kappa)$, hence the result. ■

Appendix B: Nonlinear Contracts

We allow here for general reward functions $Y(r)$, where $r \equiv b+\theta$ and Br is the employer's revenue on the verifiable task. As it will be more convenient to work directly with effort b rather than the marginal incentive $y(b) \equiv Y'(b)$, let us define the pseudo-cost function

$$\hat{C}(b) \equiv \min_{a} \{C(a,b) - va\}, \tag{B.1}$$

and denote $\hat{a}(b)$ the minimizing choice. It is easily verified that $\hat{a}'(b) < 0$ when $C_{ab} < 0$ and that $\hat{C}(b)$ is strictly convex. To preclude unbounded solutions, we shall assume that $\lim_{b \to +\infty} \hat{C}'(b) \geq B$. The allocative component of total surplus is

$$\hat{w}(b) \equiv A\hat{a}(b) + Bb - \hat{C}(b), \tag{B.2}$$

which shall take to be strictly quasiconvex and maximized at $b^* > 0$. The effort levels b_i chosen by each type $i = H, L$ are given by $\hat{C}'(b_i) = Y'(b_i+\theta_i)$, with resulting utilities are $U_i = Y(b_i+\theta_i) - \hat{C}(b_i)$, so the relevant participation and incentive constraints are now:

$$U_L = Y(b_L + \theta_L) - \hat{C}(b_L) \geq \bar{U}, \tag{B.3}$$

$$U_H \geq U_L + \hat{C}(b_L) - \hat{C}(b_L - \Delta\theta), \tag{B.4}$$

$$U_L \geq U_H - \hat{C}(b_H + \Delta\theta) + \hat{C}(b_H). \tag{B.5}$$

Summing up the last two yields $\hat{C}(b_L) - \hat{C}(b_L - \Delta\theta) \leq \hat{C}(b_H + \Delta\theta) - \hat{C}(b_H)$, which by strict convexity of $\hat{C}$ requires that $r_L \equiv b_L + \theta_L < b_H + \theta_H \equiv r_H$.

1. Monopsony. As before, the low types' participation and high type's incentive constraints must be binding in an optimum, so the firm solves

$$\max_{\{b_H, b_L\}} \left\{ \sum_{i=H,L} q_i \left[\hat{w}(b_i) + B\theta_i - \bar{U}\right] - q_H[\hat{C}(b_L) - \hat{C}(b_L - \Delta\theta)] \right\},$$

leading to $b_H^m = b^*$ and

$$\hat{w}'(b_L^m) = \frac{q_H}{q_L} \left[\hat{C}'(b_L^m) - \hat{C}'(b_L^m - \Delta\theta)\right] \tag{B.6}$$

when the solution is interior and to $b_L^m = \Delta\theta$ when it is not; in either case, $b_H^m < b^*$.

As before, the firm is willing to employ the low types provided the profits they generate exceed the rents abandoned to the high types:

$$q_L \left[\hat{w}(b_L^m) + B\theta_L - \bar{U}\right] \geq q_H \left[\hat{C}(b_L) - \hat{C}(b_L - \Delta vi\theta)\right], \tag{B.7}$$

which defines a lower bound $\underline{q}_L < 1$ for q_L. Compared with the case of linear incentives, the greater flexibility afforded by nonlinear schemes allows the monopsonist to reduce the high type's rents,

$$U_H = \bar{U} + \hat{C}(b_L^m) - \hat{C}(b_L^m - \Delta\theta) < \bar{U} + \hat{C}'(b_L^m)\Delta\theta \equiv \bar{U} + y_L^m \Delta\theta \tag{B.8}$$

while keeping the distortion –underincentivization of the low types– unchanged, thereby increasing his profits.

2. Perfect competition. In a separating competitive equilibrium (with no cross-subsidies), $U_i = w(b_i) + \theta_i B$ for $i = L, H$, and by the same argument as in Section 3.2 the low type must get his efficient symmetric-information allocation, so $b_L^c = b^*$. Furthermore, among all such allocations that are incentive-compatible, the most attractive to high types is the LCS one, defined by: .

$$b_H \equiv \arg\max_b \left\{ \hat{w}(b) \mid \hat{w}(b^*) - \hat{w}(b) \geq B\Delta\theta - \hat{C}(b + \Delta\theta) + \hat{C}(b) \right\}. \tag{B.9}$$

Denoting the Lagrange multiplier as $\lambda \geq 0$, the first-order condition is $-\hat{w}'(b)(1-\lambda) = \lambda[\hat{C}'(b + \Delta\theta) - \hat{C}'(b)]$, requiring that $b \geq b^*$ and leading to two cases:

(i) If $\hat{C}(b^* + \Delta\theta) - \hat{C}(b^*) \geq B\Delta\theta$, then $b = b^*$.

(ii) If $\hat{C}(b^* + \Delta\theta) - \hat{C}(b^*) < B\Delta\theta$, then b_H is uniquely given by

$$\hat{w}(b^*) - \hat{w}(b_H^c) = B\Delta\theta - \hat{C}(b_H^c + \Delta\theta) + \hat{C}(b_H^c), \tag{B.10}$$

since $\Omega(b) \equiv \hat{w}(b^*) - \hat{w}(b) + \hat{C}(b + \Delta\theta) - \hat{C}(b)$ is strictly increasing on $(b^*, +\infty)$, with $\Omega(b^*) < B\Delta\theta$ and $\Omega(b) \geq \hat{C}'(b) > B\Delta\theta$ for all b large enough. Competition now leads to overincentivization only if $\hat{C}(b^* + \Delta\theta) - \hat{C}(b^*)$ is not too large, meaning that b^* itself is not too large, which in turn occurs when A is large enough.[33] Whereas the greater flexibility afforded by nonlinear contracts allows a monopsonist to reduce the high types' rents and increase his profits, in a competitive industry those benefits are appropriated by the high types, in the form of more efficient contracts:

$$\hat{w}(b^*) - \hat{w}(b_H^c) \leq B\Delta\theta - \hat{C}(b_H^c + \Delta\theta) + \hat{C}(b_H^c) < B\Delta\theta - \hat{C}'(b_H^c)\Delta\theta \equiv (B - b_H^c)\Delta\theta. \tag{B.11}$$

As before, the above separating allocation is the equilibrium if and only if it is interim efficient, meaning that there is no profitable deviation consisting of lowering b_H by a small amount $\delta b_H = -\varepsilon$ to increase total surplus while offering compensating transfers $\delta Y_L = \hat{C}'(b_H + \Delta\theta) - \hat{C}'(b_H)$ to low types so as to maintain incentive compatibility (note that $\delta U_H = [Y'(b_H + \theta_H) - \hat{C}(b_H)]\varepsilon = 0$ to the first order). In other words,

$$q_H \hat{w}'_H(b_H) + q_L \left[\hat{C}'(b_H + \Delta\theta) - \hat{C}'(b_H) \right] \geq 0, \tag{B.12}$$

which defines a lower bound $\tilde{q}_L < 1$ for q_L. We denote again $q_L^* \equiv \max\{ \underline{q}_L, \tilde{q}_L \}$.

The following proposition summarizes the above results.

Proposition 18 *Let $q_L > q_L^*$. With unrestricted nonlinear contracts, it remains the case that:*

1. *A monopsonist distorts downward the measurable effort of low types: $b_L^m < b^* = b_H^m$, with b_L^m given by (B.6).*

[33] Note that $\hat{C}(b^* + \Delta\theta) - \hat{C}(b^*) < \hat{C}'(b^* + \Delta\theta)\Delta\theta$, so $\hat{C}(b^* + \Delta\theta) - \hat{C}(b^*) < B\Delta\theta$ if $b^* < \hat{C}'^{-1}(B) - \Delta\theta$. This, in turn, occurs when $A\hat{a}'[\hat{C}'^{-1}(B) - \Delta\theta] + B < \hat{C}'[\hat{C}'^{-1}(B) - \Delta\theta]$, for which it suffices that a be large enough.

2. Under perfect competition, if $\hat{C}(b^*+\Delta\theta)-\hat{C}(b^*) < B\Delta\theta$, firms distort upwards the measurable effort of high types: $b_L^c = b^* < b_H^c$, with b_H^c given by (B.10). If $\hat{C}(b^*+\Delta\theta)-\hat{C}(b^*) \geq B\Delta\theta$, both effort levels are efficient: $b_L^c = b^* = b_H^c$.

For small $\Delta\theta$, Taylor expansions show that: (i) $b^* - b_L^m$ is, as before, of order $\Delta\theta$, with coefficient q_H/q_L; (ii) under competition, since $\hat{C}'(b^*) < B\Delta\theta$, $b_H^c - b^*$ is positive and again of order $\sqrt{\Delta\theta}$, with coefficient independent of q_H and q_L; (iii) $1-\tilde{q}_L$ is also of order $\Delta\theta$. Therefore, the ratio L^c/L^m is at least of the same order as $q_H\Delta\theta/q_L\,[(q_H/q_L)\Delta\theta]^2 = (q_L/q_H)\,\Delta\theta$, or equivalently $(\Delta\theta)^{-3/2}$, and hence arbitrarily large.

3. Quadratic-cost case. Here again, the cost function (A.1) leads to explicit and transparent solutions, including for the comparison of efficiency losses under monopsony and competition. Minimizing $C(a,b) - va$ over a leads to $\hat{a}(b) = v - \gamma b$ and

$$\hat{C}(b) = \frac{1}{2}[b^2 - (v-\gamma b)^2] = \frac{1}{2}\left[(1-\gamma^2)b^2 + 2v\gamma b - v^2\right]. \tag{B.13}$$

Therefore $\hat{C}'(b) = (1-\gamma^2)b + v\gamma$ and $\hat{w}'(b) = B - \gamma\,(A+v) - (1-\gamma^2)b$, leading to

$$\begin{aligned}
(1-\gamma^2)b^* &= B - \gamma\,(A+v)\,, && \text{(B.14)}\\
\hat{w}(b^*) - \hat{w}(b) &= -w''(b^*)\frac{(b-b^*)^2}{2} = \frac{1-\gamma^2}{2}(b-b^*)^2, &&\\
\hat{C}(b) - \hat{C}(b^*) &= \hat{w}(b^*) - \hat{w}(b^*) + (B-\gamma A)\,(b-b^*) &&\\
&= \frac{1-\gamma^2}{2}(b-b^*)^2 + (B-\gamma A)\,(b-b^*). && \text{(B.15)}
\end{aligned}$$

Under *monopsony*, b_L^m is defined (when interior) by (B.6), which now becomes:

$$\begin{aligned}
B - \gamma\,(A+v) - (1-\gamma^2)b_L^m &= \frac{q_H}{q_L}(1-\gamma^2)\Delta \iff &&\\
b_L^m &= b^* - \frac{q_H}{q_L}\Delta. && \text{(B.16)}
\end{aligned}$$

This is the same outcome as with linear contracts, and the marginal incentives $y_i \equiv Y'(b_i+\theta_i) = C'(b_i)$ are also unchanged. The rent left to high types is now lower, however, implied by (B.8).

Under *competition*, b_H^c is given by (B.9). Now, for all b, $\hat{w}(b^*) - \hat{w}(b) \geq B\Delta\theta - \hat{C}(b+\Delta\theta) + \hat{C}(b_H)$ if and only if

$$\begin{aligned}
B\Delta\theta - [\hat{w}(b^*) - \hat{w}(b)] &\leq \left[\hat{C}(b_H+\Delta\theta) - C(b^*)\right] - \left[\hat{C}(b) - C(b^*)\right] \iff\\
B\Delta\theta - \frac{1-\gamma^2}{2}(b-b^*)^2 &= \frac{1-\gamma^2}{2}(b+\Delta\theta-b^*)^2 - \frac{1-\gamma^2}{2}(b-b^*)^2 + (B-\gamma A)\,\Delta\theta \iff\\
\frac{2\gamma A\Delta\theta}{1-\gamma^2} &\leq \Delta\theta\,[2(b-b^*)+\Delta\theta] + (b-b^*)^2 = (b-b^*+\Delta\theta)^2
\end{aligned}$$

Therefore (B.9) becomes

$$b_H^c = b^* + \max\left\{0, -\Delta\theta + \sqrt{\frac{2\gamma A \Delta\theta}{1-\gamma^2}}\right\} \tag{B.17}$$

or, in terms of marginal incentives, $y_H^c = y^* + \max\left\{0, -\omega + \sqrt{2\omega\gamma A}\right\}$. Comparing these expressions to (A.8), its is clear that competition leads to less distortion when nonlinear contracts are allowed (possibly none, for low A), whereas the monopsony distortion remains the same. Nonetheless, when A is high enough, or $\Delta\theta$ small enough, competition can still be efficiency-reducing:

Proposition 19 *Let $C(a,b) = a^2/2 + b^2/2 + \gamma ab$. When employers can offer arbitrary nonlinear contracts, social welfare is lower under competition than under monopsony if and only if q_L is high enough and*

$$\frac{1+q_H}{2q_L} < \left(\frac{\gamma}{1-\gamma^2}\right)\left(\frac{A}{\Delta\theta}\right). \tag{B.18}$$

Proof. We have

$$\begin{aligned} L^m &= \frac{q_L\left(b^* - b_L^m\right)^2}{2(1-\gamma^2)} < \frac{q_H\left(b_H^c - b^*\right)^2}{2(1-\gamma^2)} = L^c \iff \\ \sqrt{\frac{q_L}{q_H}}\frac{q_H}{q_L}\Delta\theta &< \sqrt{\frac{2\gamma A\Delta\theta}{1-\gamma^2}} - \Delta\theta \iff \frac{1}{2}\left(1+\sqrt{\frac{q_H}{q_L}}\right)^2 < \frac{\gamma}{1-\gamma^2}\frac{A}{\Delta\theta} \iff \\ \underline{\hat{M}}(q_L) &\equiv \frac{1+q_H}{2q_L} < \frac{\gamma}{1-\gamma^2}\frac{A}{\Delta\theta} \end{aligned}$$

Thus, together with the interim-efficiency condition, which remains unchanged, we require:

$$\underline{\hat{M}}(q_L) \equiv \frac{2-q_L}{2q_L} < \frac{\gamma}{1-\gamma^2}\frac{A}{\Delta\theta} \leq \frac{1}{2}\left(\frac{q_L}{1-q_L}\right)^2 + \frac{q_L}{1-q_L} \equiv \bar{M}(q_L).$$

While the upper bound $\bar{M}(q_L)$ is unchanged from (A.12), the lower bound is higher than in the linear-contracts case, $\underline{\hat{M}}(q_L) > \underline{M}(q_L)$,since

$$\frac{1-q_L}{2q_L} + \sqrt{\frac{1-q_L}{q_L}} < \frac{1-q_L}{2q_L} + \frac{1}{2q_L}.$$

Nonetheless, it remains the case that:

(i) $\underline{M}(q_L) < \bar{M}(q_L)$ if and only if $q_H/q_L < 1$, so for any $q_L > 1/2$, (A.12) defines a nonempty range for $(A/\Delta\theta)\left[\gamma/(1-\gamma^2)\right]$.

(ii) As $q_L \to 1$, and $\bar{M}(q_L) \to +\infty$ so arbitrary large values of $(A/\Delta\theta)\left[\gamma/(1-\gamma^2)\right]$ become feasible, including arbitrarily large values of A or arbitrarily low values of $\Delta\theta$. In particular, imposing $\gamma A < B(1-\gamma^2) - q_H\Delta\theta/q_L$ to ensure $0 < y^* < y_L^m$ is never a problem for q_L large enough. ■

Appendix C: Main Proofs

Proof of Proposition 1. Only the comparative-statics results remains to prove. First, differentiating (10) and (17) with respect to $\Delta\theta$ yields

$$\frac{\partial y_L^m}{\partial \Delta\theta} = \frac{q_H}{q_L}\frac{1}{w_{yy}(y_L^m)} < 0 < q_H \frac{w_y(y_L^m)}{-w_{yy}(y_L^m)} = \frac{\partial L^m}{\partial \Delta\theta}. \tag{C.1}$$

Turning next to A, we have

$$\frac{\partial y_L^m}{\partial A} = \frac{w_{yA}(y_L^m)}{-w_{yy}(y_L^m)} = \frac{a'(y_L^m)}{-w_{yy}(y_L^m)} < 0 \tag{C.2}$$

$$\begin{aligned}\frac{1}{q_L}\frac{\partial L^m}{\partial A} &= w_A(y^*;A,B) - w_A(y_L^m;A,B) + w_y(y^*;A,B)\frac{\partial y^*}{\partial A} - w_y(y_L^m;A,B)\frac{\partial y_L^m}{\partial A} \\ &= a(y^*) - a(y_L^m) - \frac{q_H}{q_L}\Delta\theta\frac{\partial y_L^m}{\partial A},\end{aligned} \tag{C.3}$$

showing clearly the two opposing effects discussed in the text, and which exactly cancel out in the quadratic-cost case (see (A.5)). Similarly, for B :

$$\frac{\partial y_L^m}{\partial B} = \frac{w_{yB}(y_L^m)}{-w_{yy}(y_L^m)} = \frac{b'(y_L^m)}{-w_{yy}(y_L^m)} > 0 \tag{C.4}$$

$$\begin{aligned}\frac{1}{q_L}\frac{\partial L^m}{\partial B} &= w_B(y^*;A,B) - w_B(y_L^m;A,B) + w_y(y^*;A,B)\frac{\partial y^*}{\partial B} - w_y(y_L^m;A,B)\frac{\partial y_L^m}{\partial B} \\ &= b(y^*) - b(y_L^m) - \frac{q_H}{q_L}\Delta\theta\frac{\partial y_L^m}{\partial B},\end{aligned} \tag{C.5}$$

showing again two offsetting effects on L^m. ■

Proof of Lemma 1. Denote U_L^c and U_H^c the two types' utilities in the LCS allocation, and recall that the former takes the same value as under symmetric information**.**

Claim 1 *The LCS allocation is interim efficient if and only if it solves the following program*

$$(\mathcal{P}): \max_{(U_L,U_H,y_H,y_L)} \{U_H\}, \quad \textit{subject to:}$$

$$U_L \geq U_L^c = w(y^*) + \theta_L B \tag{C.6}$$

$$U_L \geq U_H - y_H\Delta\theta \tag{C.7}$$

$$U_H \geq U_L + y_L\Delta\theta \tag{C.8}$$

$$0 \leq \sum_{i=H,L} q_i\left[w(y_i) + B\theta_i - U_i\right], \tag{C.9}$$

Proof. Conditions (C.7) and (C.8) are the incentive constraints for types L and H respectively, and condition (C.9) the employers' interim break-even constraint. Now, note that:

(i) If the LCS allocation does not achieve the optimum, interim efficiency clearly fails.

(ii) If the LCS allocation solves ($\mathcal{P}$), there can clearly be no incentive-compatible Pareto improvement in which $U_H > U_H^c$, but neither can there be one in which $U_H = U_H^c$ and $U_L > U_L^c$. Otherwise, note first that one could without loss of generality take such an allocation, to satisfy $y_H \geq y^*$; otherwise, replacing y_H by y^* while keeping U_H and U_L unchanged strictly increases profits, so the LCS allocation remains (even more) dominated. Starting from such an allocation with $y_H \geq y^*$, let us now reduce U_L by some small $\eta > 0$ and increase U_H by some small ε, while also increasing y_H by $(\varepsilon + \eta)/\Delta\theta$ to leave (C.6) unchanged (while (C.7) is only strengthened). This results in extra profits of $q_H [w'(y_H)(\varepsilon + \eta)\Delta\theta - \varepsilon] + q_L \eta$, which is positive as long as

$$\eta > \frac{q_H [1 - w'(y_H)\Delta\theta] \varepsilon}{q_L + q_H w'(y_H)\Delta\theta}.$$

Both types and the firm are now strictly better off than in the LCS allocation, contradicting the fact that it is a solution to ($\mathcal{P}$). ■

To study interim efficiency and prove Lemma 1, let us therefore analyze the solution(s) to ($\mathcal{P}$). First, condition (C.9) must be binding, otherwise U_L and U_H could be increased by the same small amount without violating the other constraints. Second, (C.7) must also be binding. Indeed, solving ($\mathcal{P}$) without that constraint leads to $U_L = U_L^c$ and $y_H = y^*$; maximizing U_H subject to (C.9) then leads to $y_L = y^*$ and $U_L = w(y^*) + B\theta_H$, so (C.6) holds with equality, a contradiction. Third, (C.8) now reduces to $y_H \geq y_L$. Two cases can then arise:

(i) If (C.6) is binding, the triple $(U_L,\, U_H,\, y_H)$ is uniquely given by the same three equality constraints as the LCS allocation, and thus coincides with it.

(ii) If (C.6) is not binding, the solution to ($\mathcal{P}$) is the same as when that constraint is dropped. Substituting (C.7) into (C.9), and both being equalities, we have $U_H = \Sigma q_i [w(y_i) + B\theta_i] + q_L y_H \Delta\theta$, so ($\mathcal{P}$) reduces to

$$\max_{y_H, y_L} \{q_H w(y_H) + q_L [w(y_L) + y_H \Delta\theta] \;\; |y_H \geq y_L\}. \tag{C.10}$$

For all $x \geq 0$, define the function $\tilde{y}(x) \equiv \arg\max_y \{w(y) + xy\}$ and let $\bar{x} \equiv -w'(B)$. On the interval $[0, \bar{x}]$ the function $\tilde{y}$ is given by $w'(\tilde{y}(x)) = -x$, so it is strictly increasing up to $\tilde{y}(\bar{x}) = B$, while for $x > \bar{x}$, $\tilde{y}(x) \geq B$. Furthermore, it is clear that $\tilde{y}(x) \geq y^*$ with equality only at $x = 0$, so the pair $(y_H = \tilde{y}(q_L \Delta\theta / q_H), y_L = y^*)$ is the solution to (C.10). It is then indeed the case that (C.6) is non-binding, $U_L > U_L^c = U_H - y_H \Delta\theta$, if and only if

$$w(y^*) + \theta_L y^* < q_H [w(y_H) + B\theta_H] + q_L [w(y^*) + B\theta_L] - q_H y_H \Delta\theta$$

or equivalently $H(q_L \Delta\theta / q_H) \geq 0$, where

$$H(x) \equiv w(\tilde{y}(x)) - w(y^*) + [B - \tilde{y}(x)] \Delta\theta \geq 0. \tag{C.11}$$

Note that $H(0) > 0$ and $H(x) < 0$ for $x \geq \bar{x}$, while on $[0, \bar{x}]$ we have $H'(x) = [w'(\tilde{y}(x)) - \Delta\theta] \tilde{y}'(x) = (q_L/q_H - 1) \Delta\theta \tilde{y}'(x) = -(\Delta\theta / q_H) \tilde{y}'(x)$. Therefore, there exists a unique $\tilde{x} \in (0, \bar{x})$ such that (C.6) is non-binding –and the solution to ($\mathcal{P}$) thus differs from the LCS allocation– if and only if $\Delta\theta q_L / q_H <$

$\tilde{x}$. Equivalently, the LCS allocation is the unique solution to $(\mathcal{P})$, and therefore interim efficient, if and only if $q_L/(1-q_L) \geq \tilde{x}/\Delta\theta \equiv \tilde{q}_L/(1-\tilde{q}_L)$, hence the result. For small $\Delta\theta$ it is easily verified from $H(\tilde{x}) \equiv 0$ and $w'(\tilde{y}(x)) = -x$ (implying $\tilde{y}'(x) = -1/w''(\tilde{y}(x))$) that $-\tilde{x}^2/w''(y^*) \approx 2(B-y^*)\Delta\theta$, so that $\tilde{q}_L \approx 1 - \chi\sqrt{\Delta\theta}$, where $1/\chi \equiv \sqrt{-2w"(y^*)(y^*-B)}$. ■

Proof of Proposition 2. To complete the proof of Results (1) and (2), it just remains to show that: (a) If the LCS allocation is interim efficient, it is a competitive equilibrium; (b) It is then the unique one. We shall also prove here that: (c) If the LCS allocation is not interim efficient, there exists no competitive equilibrium in pure strategies.

Claim 2 *In any competitive equilibrium, the utilities (U_L, U_H) must satisfy*

$$U_L \geq U_L^c = U_L^{SI} \equiv w(y^*) + \theta_L B, \tag{C.12}$$

$$U_H \geq U_H^c \equiv w(y_H^c) + \theta_H B, \tag{C.13}$$

Proof. If $U_L < U_L^c$, a firm could offer the single contract $(y = y^*,\ z = z_L^c - \varepsilon)$ for ε small, attracting and making a profit ε on type θ_L (perhaps also attracting the more profitable type θ_H). Similarly, if $U_H < U_H^c$, it could offer the incentive-compatible menu $\{(y^*, z_L^c - \varepsilon),\ (y_H^c, z_H^c - \varepsilon)\}$ thereby attracting and making a profit ε on type θ_H (perhaps also attracting and making zero profit on type θ_L). ■

Claim 3 *If an allocation Pareto dominates (in the interim-efficiency sense) the least-cost separating one, it must involve a cross-subsidy from high to low types, meaning that*

$$w(y_H) + B\theta_H - U_H > 0 > w(y_L) + B\theta_L - U_L, \tag{C.14}$$

Proof. If $U_L \leq w(y_L) + \theta_L B$, then $U_L \geq U_L^c$ requires that $y = y^*$ and $U_L = w(y_L) + \theta_L B$; the break-even condition then requires that $w(y_H) + \theta_H B \geq U_H$, and Pareto-dominance that $U_H > U_H^c$. Therefore $w(y_H) + \theta_H B > U_H^c = U_L + y_H\Delta\theta = w(y^*) + B\theta_L + y_H\Delta\theta$, or finally $w(y_H) < w(y^*) - (B - y_H)\Delta\theta$. By (22) this means $y_H > y_H^c$, but then $w(y_H) + \theta_H B < w(y_H^c) + \theta_H B = U_H^c$, a contradiction. Thus, $w(y_L) + B\theta_L - U_L < 0$, meaning that low types get more than the total surplus they generate. For the employer to break even, it must be that high types get strictly less, $w(y_H) + B\theta_H - U_H > 0$. ■

We are now ready to establish the properties (a)-(c) listed above, and thereby complete the proof of Results (1) and (2) in Proposition 2.

(a) Suppose that the LCS allocation, defined by (18)-(22), is offered by all firms. Could another one come in and offer a different set of contracts, leading to new utilities $(U_L,\ U_H)$ and a strictly positive profit? First, note that we can without loss of generality assume that $U_L \geq U_L^c$: if $U_L < U_L^c$ and U_H is indeed selected (with positive probability) by type H, then $(U_L^c\ ,\ U_H)$ is incentive-compatible. By offering U_L^c to L types (via their symmetric-information allocation), the deviating firm does not alter its profitability. Second, if $U_H < U_H^c$, the deviating employer does not attract type H; since it cannot make money on type L while providing $U_L \geq U_L^c$, the deviation is not profitable. Finally, suppose that $U_L \geq U_L^c$ and $U_H \geq U_H^c$. If at least one inequality is strict, then interim efficiency of the LCS allocation implies that the deviating firm loses money. If both are equalities, let us specify (for instance) that both types workers, being indifferent, do not select the deviating firm.

(b) By (C.12)-(C.13), in any equilibrium both types must be no worse off than in the LCS allocation, and similarly for the firm, which must make non-negative profits. If any of these inequalities is strict there is Pareto dominance, so when the LCS allocation is interim efficient, they must all be equalities, giving the LCS allocation as the unique solution.

(c) Suppose now that LCS allocation is not interim efficient. The contract that solves $(\mathcal{P})$ is then such that $y_H = \tilde{y}(x), y_L = y^*$, $U_L > U_L^c$ (equation (C.6) is not binding) and $U_H > U_H^c$ (since the LCS does not solve $(\mathcal{P})$). A firm can then offer a contract with the same y_H and y_L but reducing both U_H and U_L by the same small amount, resulting in positive profits; the LCS allocation is thus not an equilibrium. Suppose now that some other allocation, with utilities U_L and U_H, is an equilibrium. As seen in (b), it would have to Pareto-dominate the LCS allocation, which by Claim 3 implies:

$$\begin{aligned} &U_L \geq U_H - y_H \Delta\theta, \\ &w(y_H) + B\theta_H - U_H > 0. \end{aligned}$$

Consider now a deviating employer offering a single contract, aimed at the high type: $y_H' = y_H + \varepsilon$ and $U_H' = U_H + (\varepsilon\Delta\theta)/2 < w(y_H') + B\theta_H$. The low type does not take it up, as it would yield $U_L = U_L^c - (\varepsilon\Delta\theta)/2$. The high type clearly does, leading to a positive profit for the deviator.‖

The only part of Proposition 2 remaining to prove are the comparative static results. Differentiating (22) and (24) with respect to $\Delta\theta$ yields

$$\frac{\partial y_H^c}{\partial \Delta\theta} = \frac{B - y_H^c}{\Delta\theta - w_y(y_H^c; A, B)} > 0, \quad \frac{\partial L^c}{\partial \Delta\theta} = -q_H w_y(y_H^m; A, B)\frac{\partial y_H^c}{\partial \Delta\theta} > 0. \tag{C.15}$$

Turning next to A,

$$\begin{aligned} -\Delta\theta \frac{\partial y_H^c}{\partial A} &= w_A(y^*; A, B) - w_A(y_H^c; A, B) + w_y(y^*; A, B)\frac{\partial y^*}{\partial A} - w_y(y_H^c; A, B)\frac{\partial y_H^c}{\partial A} \\ &= a(y^*) - a(y_H^c) - w_y(y_H^c; A, B)\frac{\partial y_H^c}{\partial A} \Rightarrow \\ \frac{\partial y_H^c}{\partial A} &= \frac{a(y_H^c) - a(y^*)}{\Delta\theta - w_y(y_H^c; A, B)} < 0 < -q_H \Delta \frac{\partial y_H^c}{\partial A} = \frac{\partial L^c}{\partial A}. \end{aligned} \tag{C.16}$$

Again there is a direct and an indirect effect of A on L^c, but now the direct one always dominates. For B, in contrast, the ambiguity remains

$$\begin{aligned} \Delta\theta - \Delta\theta \frac{\partial y_H^c}{\partial B} &= w_B(y^*; A, B) - w_B(y_H^c; A, B) + w_y(y^*; A, B)\frac{\partial y^*}{\partial B} - w_y(y_H^c; A, B)\frac{\partial y_H^c}{\partial B} \\ &= b(y^*) - b(y_H^c) - w_y(y_H^c; A, B)\frac{\partial y_H^c}{\partial B} \Rightarrow \end{aligned}$$

$$\frac{\partial y_H^c}{\partial B} = \frac{\Delta\theta + b(y_H^c) - b(y^*)}{\Delta\theta - w_y(y_H^c; A, B)} > 0, \tag{C.17}$$

$$\frac{1}{q_H \Delta\theta}\frac{\partial L^c}{\partial B} = 1 - \frac{\partial y_H^c}{\partial B} = \frac{-w_y(y_H^c; A, B) - b(y_H^c) + b(y^*)}{\Delta\theta - w_y(y_H^c; A, B)}. \tag{C.18}$$

In the quadratic case, L^c is independent of B and the last term thus equal to zero; see (A.9). ■

Proof of Proposition 4. We solve for the symmetric equilibrium under the assumption that market shares are always interior, and thus given by (31). In Appendix D we verify that individual deviations to corner solutions (one firm grabbing the whole market for some worker type, or on the contrary dropping them altogether) can indeed be excluded.

To characterize the symmetric solution to (32)-(35), we distinguish three regions.

Region I. Suppose first that the low type's individual rationality constraint is not binding, $U_L > \bar{U}$, so that $\nu = 0$.

Lemma 3 *If $\nu = 0$, then $\mu_H = 0 \leq \mu_L$ and $y_L = y^* \leq y_H$.*

Proof. (i) If $\mu_H = \mu_L = 0$, then $y_H = y_L = y^*$ by (38)-(39), so (33)-(34) imply that $U_H - U_L = y^*\Delta\theta$. Next, from (36)-(37) we have $m_H - m_L = t - m_L = 0$, whereas $m_H - m_L \equiv B\Delta\theta - (U_H - U_L) = (B - y^*)\Delta\theta > 0$, a contradiction.

(ii) If $\mu_H > 0 = \mu_L$ condition (39) implies $w'(y_L) > 0$, hence $y_L < y^*$, and condition (38) $y_H = y^*$. Moreover, (36)-(37) and $\mu_H > \mu_L$ require that $m_H < t < m_L$. However,

$$m_H - m_L = w(y^*) - w(y_L) + (B - y_L)\Delta\theta > 0,$$

a contradiction. We are thus left with $\mu_H = 0 < \mu_L$, which implies $y_L = y^* < y_H$ by (39) and (38) respectively. ■

Let us now derive and characterize y_H as a function of t. We can rewrite (38) as

$$tq_H w'(y_H) = -\mu_L \Delta\theta = -q_L(m_L - t)\Delta\theta. \tag{C.19}$$

Combining (36)-(37) and recalling that $m_i \equiv w(y_i) + \theta_i B - U_i$ yields

$$\begin{aligned} U_L + t &= q_H\left[w(y_H) + \theta_H B - (U_H - U_L)\right] + q_L\left[w(y^*) + \theta_L B\right] \\ &= q_H\left[w(y_H) + \theta_H B - y_H\Delta\theta\right] + q_L\left[w(y^*) + \theta_L B\right], \end{aligned} \tag{C.20}$$

where the second equality reflects the fact that (33) is an equality, since $\mu_L > 0$. Therefore:

$$\begin{aligned} m_L - t &= w(y^*) + \theta_L B - U_L - t \\ &= w(y^*) + \theta_L B - q_L\left[w(y^*) + \theta_L B\right] - q_H\left[w(y_H) + \theta_H B - y_H\Delta\theta\right] \\ &= q_H\left[w(y^*) - w(y_H) - (B - y_H)\Delta\theta\right] \end{aligned} \tag{C.21}$$

Substituting into (C.19) yields

$$\Phi(y_H;t) \equiv w(y_H) - w(y^*) + (B - y_H)\Delta\theta + \frac{tw'(y_H)}{q_L\Delta\theta} = 0. \tag{C.22}$$

The following lemma characterizes the equilibrium value of y_H over Region I, denoted $\hat{y}_H^I(t)$.

Lemma 4 *For any $t \geq 0$ there exists a unique $\hat{y}_H^I(t) \in (y^*, B)$ to (C.22). It is strictly decreasing in t, starting from the perfectly competitive value $\hat{y}_H^I(0) = y_H^c$.*

Proof. The function $\Phi(y;t)$ is strictly decreasing in y on $[y^*, B)$, with $\Phi(y^*) > 0 > \Phi(B)$, hence existence and uniqueness. Strict monotonicity then follows from the fact that Φ is strictly decreasing in t, while setting $t = 0$ in (C.22) shows that $\hat{y}_H^I(0)$ must equal y_H^c, defined in (22) as the unique solution to

$$w(y^*) - w(y_H^c) = (B - y_H^c)\Delta\theta.$$

It only remains to verify that the solution $\hat{y}_H^I(t)$ is consistent with the initial assumption that $\nu = 0$, or equivalently $U_L > \bar{U}$. By (C.20), we have for all y_H

$$\begin{aligned} U_L + t &= q_H\left[w(y_H) + \theta_H B - y_H\Delta\theta\right] + q_L\left[w(y^*) + \theta_L B\right] \\ &= w(y^*) + \theta_L B + q_H\left[(B - y_H)\Delta\theta + w(y_H) - w(y^*)\right]. \; \blacksquare \end{aligned}$$

For $y_H = \hat{y}_H^I(t)$, the corresponding value of U_L is strictly above $\bar{U}$ if and only if $\psi(t) > \bar{U} + t$, where we define for all t:

$$\psi(t) \equiv w(y^*) + \theta_L B + q_H\left[\left(B - \hat{y}_H^I(t)\right)\Delta\theta - w(y^*) + w(\hat{y}_H^I(t))\right]. \tag{C.23}$$

Lemma 5 *There exists a unique $t_1 > 0$ such that $\psi(t) \geq \bar{U} + t$ if and only if $t \leq t_1$. On $[0, t_1]$, the low type's utility U_L is strictly decreasing in t, reaching $\bar{U}$ at t_1.*

Proof. At $t = 0$ the bracketed term is zero by definition of $\hat{y}_H^I(0) = y_H^c$, so $\psi(0) = w(y^*) + \theta_L B > \bar{U}$ by (16), which stated that a monopsonist hires both types, and $\lim_{t\to+\infty}\left[\psi(t) - \bar{U} - t\right] = -\infty$, there exists at least one solution to $\psi(t) = \bar{U} + t$. To show that it is unique and the monotonicity of U_L, we establish that, $\psi'(t) < 1$ for all $t > 0$. From (C.22) and (C.23), this means that

$$\begin{aligned} q_H\left[\Delta\theta - w'(\hat{y}_H)\right]\left(\frac{-w'(\hat{y}_H)/q_L\Delta\theta}{\Delta\theta - w'(\hat{y}_H) - tw''(\hat{y}_H)/q_L\Delta\theta}\right) &< 1 \iff \\ q_H\left[\Delta\theta - w'(\hat{y}_H)\right]\left(-w'(\hat{y}_H)/q_L\Delta\theta\right) &< \Delta\theta - w'(\hat{y}_H) - tw''(\hat{y}_H)/q_L\Delta\theta \iff \\ q_H\left[\Delta\theta - w'(\hat{y}_H)\right]\left(-w'(\hat{y}_H)\right) &< q_L\Delta\theta\left[\Delta\theta - w'(\hat{y}_H)\right] - tw''(\hat{y}_H) \iff \\ tw''(\hat{y}_H) &< \left[\Delta\theta - w'(\hat{y}_H)\right]\left[q_H w'(\hat{y}_H) + q_L\Delta\theta\right] \end{aligned}$$

where we abbreviated $\hat{y}_H^I(t)$ as $\hat{y}_H$. In the last expression, the left-hand side is always non-negative, whereas on the right hand side $y^* < \hat{y}_H < \hat{y}_H^c$ implies that $w'(\hat{y}_H) < 0$ and $q_H w'(\hat{y}_H) + q_L\Delta\theta > q_H w'(\hat{y}_H^c) + q_L\Delta\theta > 0$, by (23). $\blacksquare$

In summary, Region I consists of the interval $[0, t_1]$, where t_1 is uniquely defined by $\psi(t_1) = t_1$. Over that interval, $y_L = y^*$ while $y_H = \hat{y}_H^I(t)$ is strictly decreasing in t, and therefore so is the high type's relative rent, $U_H - U_L = \hat{y}_H^I(t)\Delta\theta$. The low type's utility level U_L need not be declining, but its starts at a positive value and reaches $\bar{U}$ exactly at t_1.

For $t \geq t_1$, the constraint $U_L \geq \bar{U}$ is binding. Recalling that $\mu_H \mu_L$ must always equal zero, we distinguish two subregions, depending on whether $\mu_H = 0$ (Region II) or $\mu_L = 0$ (Region III), and show that these are two intervals, respectively $[t_1, t_2]$ and $[t_2, +\infty)$, with $t_1 < t_2$. Thus, inside Region II the low type's incentive constraint is binding but not the high type's ($\mu_L > 0 = \mu_H$ for $t \in (t_1, t_2)$), whereas inside Region 2 it is the reverse ($\mu_H > 0 = \mu_L$ for $t > t_2$).

Region II. Consider first the values of t where $\mu_H = 0 < \mu_L$. As before, this implies that $y_L = y^* < y_H$ and $U_H - U_L = y_H \Delta\theta$, or $U_H = \bar{U} + y_H \Delta\theta$ since $U_L = \bar{U}$. Therefore:

$$\mu_L = q_H(m_H - t) = q_H\left[w(y_H) + \theta_H B - U_H - t\right] = q_H\left[w(y_H) + \theta_H B - y_H \Delta\theta - \bar{U} - t\right]. \quad \text{(C.24)}$$

Substituting into condition (38), the latter becomes

$$\Gamma(y_H; t) \equiv w(y_H) + \theta_H B - y_H \Delta\theta - \bar{U} - t + \frac{t w'(y_H)}{\Delta\theta} = 0. \quad \text{(C.25)}$$

On the interval $[y^*, B)$, the function $\Gamma(y; t)$ is strictly decreasing in y_H and t, with

$$\begin{aligned}
\Gamma(\hat{y}_H^I(t); t) &\equiv w(\hat{y}_H(t)) + \theta_H B - \hat{y}_H^I(t)\Delta\theta + \frac{t w'(\hat{y}_H^I(t))}{\Delta\theta} - \bar{U} - t \\
&= w(y^*) + \theta_L B + \left(1 - \frac{1}{q_L}\right) \frac{t w'(y_H^I(t_1))}{\Delta\theta} - \bar{U} - t \\
&= w(y^*) + B\theta_L - t\left(1 + \frac{q_H}{q_L}\frac{w'(y_H)}{\Delta\theta}\right) - \bar{U}. \qquad \text{(C.26)}
\end{aligned}$$

At $t = t_1$, substituting (C.22) into (C.23) yields $\Gamma(\hat{y}_H^I(t_1); t_1) = 0$. Furthermore, as t rises above t_1, $\hat{y}_H^I(t)$ decreases, so $w'(\hat{y}_H^I(t))$ increases. Since

$$q_L \Delta\theta + q_H w'(\hat{y}_H(t)) > q_L \Delta\theta + q_H w'(\hat{y}_H(0)) = q_L \Delta\theta + q_H w'(y_H^c) > 0$$

by (23), $t\left[q_L \Delta\theta + q_H w'(\hat{y}_H^I(t))\right]$ is also increasing in t, implying that $\Gamma(\hat{y}_H^I(t); t)$ is decreasing in t and therefore negative over $(t_1, +\infty)$. Next, observe that $\Gamma(y^*; t) = w(y^*) + \theta_H(B - y^*) + \theta_L y^* - \bar{U} - t$. Define therefore

$$t_2 \equiv w(y^*) + \theta_H(B - y^*) + \theta_L y^* - \bar{U}, \quad \text{(C.27)}$$

and note that

$$\begin{aligned}
t_1 &= w(y^*) + \theta_L B + q_H\left[\left(B - \hat{y}_H^I(t_1)\right)\Delta\theta - w(y^*) + w(\hat{y}_H^I(t_1))\right] - \bar{U} \\
&< w(y^*) + \theta_L B + q_H\left(B - \hat{y}_H^I(t_1)\right)\Delta\theta - \bar{U} \\
&< w(y^*) + \theta_L B + (B - y^*)\Delta\theta - \bar{U} = t_2.
\end{aligned}$$

Lemma 6 *For all* $t \in [t_1, t_2]$, *there exists a unique* $\hat{y}_H^{II}(t) \in [y^*, \hat{y}_H^I(t_1))]$ *such that* $\Gamma(\hat{y}_H^{II}(t); t) = 0$. *Furthermore,* $\hat{y}_H^{II}(t)$ *is strictly decreasing in* t, *starting at* $\hat{y}_H^{II}(t_1) = \hat{y}(t_1)$ *and reaching* y^* *at* $t = t_2$. *For all* $t > t_2$, $\Gamma(y_H; t) < 0$ *over all* $y_H \geq y^*$.

Proof. For $t \in [t_1, t_2]$ we have shown that $\Gamma(\hat{y}_H(t_1); t) \leq 0 \leq \Gamma(y^*; t)$, with the first equality strict except at t_1 and the second one strict except at t_2. Since $\Gamma(y; t)$ is strictly decreasing in y_H and t, the claimed results follow. The fact that on $(t_1, t_2]$ the graph of $\hat{y}_H^{II}(t)$ lies strictly below that of $\hat{y}_H^I(t)$ also means that if there is a kink between the two curves at t_1 it is a convex one, as illustrated on Figure III. And indeed, differentiating (C.22) and (C.25), we have$-\left(\hat{y}_H^I\right)'(t_1) < -\left(\hat{y}_H^{II}\right)'(t_1)$ if and only if

$$\begin{aligned}
-\frac{-w'}{q_L\Delta\theta(\Delta\theta - w') - tw''} &< \frac{\Delta\theta - w'}{\Delta\theta(\Delta\theta - w') - tw''}) \iff \\
q_L\Delta\theta\left(\frac{\Delta\theta}{-w'} + 1\right) + t\left(\frac{-w"}{-w'}\right) &> \Delta\theta + t\left(\frac{-w"}{\Delta\theta - w'}\right) \iff \\
t\left(-w"\right)\left[\frac{1}{-w'} - \frac{1}{\Delta\theta - w'}\right] &> \left[q_H\left(-w'\right) - q_L\Delta\theta\right]\Delta\theta = \frac{-\left(q_H w' + q_L\Delta\theta\right)}{-w'} \\
t\left(\frac{-w"}{-w'}\right)\left[\frac{1}{\Delta\theta - w'}\right] &> -\left(q_H w' + q_L\Delta\theta\right).
\end{aligned}$$

with all derivatives evaluated at $\hat{y}_H^I(t_1) = \hat{y}_H^{II}(t_1)$. Since $y^* < \hat{y}_H^I(t_1) < y_H^c$ the term on the left is positive and that on the right negative. ■

As to t_2, note that it is the only point where $\mu_H = 0 = \mu_L$ (the only intersection of Regions II and III). Indeed, this require $y_H = y^* = y_L$ by (38)-(39) and condition (39) together with $U_L = \bar{U}$ then implies that $t = m_L = w(y^*) + \theta_L B - y^*\Delta\theta_L - \bar{U} = t_2.$

Region II thus consists of the interval $[t_1, t_2]$. Over that interval, $y_L = y^*$ while $y_H = \hat{y}_H^{II}(t)$ is strictly decreasing in t, and therefore so is the high type's utility, $U_H = \bar{U} + y_H^*(t)\Delta\theta$, while the low type's utility remains fixed at $U_L = \bar{U}$. Furthermore, we can show.

Putting together Regions I and II, we shall define:

$$\hat{y}_H(t) = \begin{cases} \hat{y}_H^I(t) & \text{for } t \in [0, t_1] \\ \hat{y}_H^{II}(t) & \text{for } t \in [t_1, t_2] \end{cases}. \tag{C.28}$$

Region III. Inside this region, namely for $t > t_2$, we have $U_L = \bar{U}$ but now $\mu_H > \mu_L = 0$. This implies that $y_H = y^* > y_L$ by (38)-(39) and $U_H = \bar{U} + y_L\Delta\theta$ by (33). Furthermore,

$$\mu_H = q_H\left(t - m_H\right) = q_H\left[t + \bar{U} + y_L\Delta\theta - w(y^*) - \theta_H B\right]$$

Substituting into condition (39), the latter becomes

$$\Lambda(y_L; t) \equiv q_H\left[w(y^*) + \theta_H B - y_L\Delta\theta - \bar{U} - t\right] + \frac{tq_L w'(y_L)}{\Delta\theta} = 0. \tag{C.29}$$

On the interval $[0, y^*]$, the function $\Lambda(y;t)$ is strictly decreasing in y_L, with

$$\Lambda(y^*;t) = q_H \left[w(y^*) + \theta_H B - y^* \Delta\theta - \bar{U} - t\right] = q_H \left(t_2 - t\right) < 0.$$

Recall now that the monopsony price y_L^m is uniquely defined by $w'(y_L^m) = (q_H/q_L)\,\Delta\theta$. Therefore:

$$\Lambda(y_L^m;t) = q_H \left[w(y^*) + \theta_L B - \bar{U} + (B - y_L^m \Delta)\theta\right] > 0.$$

Lemma 7 *For all $t \geq t_2$ there exists a unique $\hat{y}_L(t)$ such that $\Lambda(\hat{y}_L(t);t) = 0$, and $y_L^m < \hat{y}_L(t) \leq y^*$, with equality at $t = t_2$. Furthermore, $\hat{y}_L(t)$ is strictly decreasing in t and $\lim_{t\to+\infty} \hat{y}_L(t) = y_L^m$.*

Proof. Existence and uniqueness have been established. Next, $\partial\Lambda(y;t)/\partial t = q_L w'(y)/\Delta\theta - 1$. At $y = \hat{y}_L(t)$, this equals $1/t$ times

$$-q_H \left[w(y^*) + \theta_H B - y_L\Delta\theta - \bar{U} - t\right] - t = -q_L t - q_H \left[w(y^*) + \theta_H B - \bar{U} - y_L\Delta\theta\right] < 0,$$

so the function $\hat{y}_L(t)$ is strictly decreasing in t. Taking limits in (C.29) as $t \to +\infty$, finally, yields as the unique solution $\lim_{t\to+\infty} \hat{y}_L(t) = y_L^m$. ■

Proof of Proposition 6. The fact that $\partial U_L/\partial t < 0$ over Region I was shown in Lemma 5. To show the last result, note that over Region III, we have

$$\begin{aligned} 2\Pi &= q_H[w(y^*) + \theta_H B - \hat{y}_L\Delta\theta] + q_L[w(\hat{y}_L) + \theta_L B] - \bar{U} \Rightarrow \\ \frac{2}{q_L}\frac{\partial\Pi}{\partial\hat{y}_L} &= w'(\hat{y}_L) - \frac{q_H}{q_L}\Delta\theta > w'(\hat{y}_L^m) - \frac{q_H}{q_L}\Delta\theta = 0, \end{aligned}$$

so profits fall as t declines, as was shown to be the case over Regions I and II. ■

Proof of Proposition 7. The result for $Y_H - Y_L$ was shown in the text. For performance-based pay, we have

$$\frac{\partial([b(y_H) + \theta_H]\, y_H - [b(y_L) + \theta_L]\, y_L)}{\partial t} = \left[b(y_H) + \theta_H + y_H b'(y_H)\right] \frac{\partial y_H}{\partial t} - \left[b(y_L) + \theta_L + y_L b'(y_L)\right] \frac{\partial y_L}{\partial t}.$$

In Regions I and II the first term is negative and the second zero; in Region III it is the reverse. Turning finally to fixed wages,

$$z_H - z_L = U_H - U_L - \theta_H y_H + \theta_L y_L - u(y_H) + u(y_L)$$

In Regions I and II, $z_H - z_L = -\theta_L(y_H - y^*) - u(y_H) + u(y^*)$ is decreasing in y_H, hence increasing in t. In Region III, $z_H - z_L = (y_L - y^*)\,\theta_H - u(y^*) + u(y_L)$, so the opposite holds. ■

Proof of Proposition 8. *Existence.* Suppose that all firms offer the package $(y^*, z^* \equiv \pi(y^*) + (B - y^*)\bar{\theta})$. A firm deviating to $(y = y^*, z)$ would not attract anyone if $z < z^*$, and attract everyone but lose money if $z > z^*$. Consider therefore a deviating offer (y, z) with $y < y^*$, hence $z > z^*$ (otherwise, it will attract no one). If (y, z) is preferred to (y^*, z^*) by both types, and strictly so for at least one type, then it must lose money, by definition of the first best. Suppose now that (y,z) is weakly preferred to (y^*,z^*) for one type, but strictly dominated for the other type. Since $\partial^2 U/\partial\theta\partial y > 0$, it is the low type who will come. The deviating firm must offer them at least $U_L^* = w(y^*) + B\bar{\theta} - y^*(\bar{\theta}-\theta_L)$, but then its profits are at most $w(y) - w(y^*) + (y^* - B)(\bar{\theta}-\theta_L) < 0$, so the deviation loses money.

Uniqueness. Suppose there is another equilibrium, and denote by (y_i, z_i) the contract chosen in it by type $i = H, L$, with resulting utility U_i. Incentive compatibility requires that $U_L \geq U_H - y_H\Delta\theta$, hence $U_H - U_L \leq y_H\Delta\theta \leq y^*\Delta\theta = U_H^* - U_L^*$, where a "*" refers to the first-best allocation. If $U_H \geq U_H^*$, this implies that $U_L \geq U_L^*$, and the only such allocation is the first best. If $U_H < U_H^*$, a deviating firm can offer $(y^*, z^* - \varepsilon)$, which for ε small enough attracts all types. Regardless of how many low types it may attract, this contract makes profits of at least $\varepsilon q_H > 0$, so the original allocation could not have been an equilibrium. ■

Proof of Proposition 9. Consider a cap at $\bar{y} \geq y^*$, which thus binds only on high types. Let us look for a least-cost separating equilibrium in which the two types are separated and firms make zero profit: the low type receives $U_L^{SI} \equiv w(y^*) + B\theta_L$ (where "SI" stands for "symmetric information") and the high type $U_H = w(\bar{y}) + B\theta_H - (1-\lambda_H)\zeta_H$, where ζ_H is the amount of inefficient transfer he is given. The latter is given by the binding incentive-compatibility condition $U_L^* = U_H - (\bar{y}\Delta\theta + \zeta_H\Delta\lambda)$, hence

$$(1-\lambda_L)\zeta_H = (B-\bar{y})\Delta\theta - [w(y^*) - w(\bar{y})] > 0, \tag{C.30}$$

since the right-hand side is decreasing in $\bar{y} \geq y^*$, and and equal to 0 at y_H^c. Thus, we have ("r" stands for "regulated")

$$U_H^r = w(y^*) + B\theta_H - \left(\frac{1-\lambda_H}{1-\lambda_L}\right)(B-\bar{y})\Delta\theta - \frac{\Delta\lambda}{1-\lambda_L}[w(y^*) - w(\bar{y})] \tag{C.31}$$

As before, this LCS allocation is the (unique) equilibrium outcome if and only if it is interim efficient. To determine when that is the case, consider the following (relaxed) program:

$$(\mathcal{P}^r): \max_{\{(0\leq y_i\leq\bar{y},\ 0\leq\zeta_i,\ U_i)\}_{i=H,L}} \{U_H\}, \quad \text{subject to:}$$

$$U_L \geq U_L^{SI} \qquad (\nu) \tag{C.32}$$

$$U_L \geq U_H - y_H\Delta\theta - \zeta_H\Delta\lambda \qquad (\xi) \tag{C.33}$$

$$0 \leq \sum_{i=H,L} [w(y_i) + \theta_i B - (1-\lambda_i)\zeta_i - U_i], \qquad (\mu) \tag{C.34}$$

Optimality clearly requires (C.34) to be binding. This in turn implies that $\zeta_L = 0$ and $y_L = y^*$, otherwise (C.34) can be relaxed without affecting any other constraint. It also cannot be that $y_H < \bar{y}$, otherwise raising it slightly relaxes (C.34) while not violating (C.33), since $-q_H w'(y_H) > -q_H w'(y^*) > y_H\Delta\theta$, by (23). Next, (C.33) must also be binding, otherwise one can find $\delta U_H > 0 > \delta U_L$ that remain feasible, violating optimality.

Suppose now that (C.32) is not binding, so $U_L > U_L^{SI}$, and $\nu = 0$. The first-order conditions in U_H and U_L are then respectively $1 - \mu q_H - \xi = 0$ and $-\mu q_L + \xi = 0$, hence $\mu = 1$ and $\xi = q_L$. Finally, the optimality condition in ζ_H is $q_L \Delta\lambda - (1 - \lambda_H) q_H \leq 0$. Therefore, if

$$q_L(\Delta\lambda) > (1 - \lambda_H) q_H, \tag{C.35}$$

it must be that the constraint $U_L = U_L^{SI}$ is in fact binding. The LCS allocation is then interim efficient, and therefore the unique equilibrium.

Finally, we consider how welfare varies with the bonus cap $\bar{y} \in [y^*, y_H^c]$. Profits always equal zero and low types always receive their symmetric-information utility U_L^{SI}. As to high types, they now achieve U_H^r, given by (C.31). As a function of $\bar{y}$, the right-hand side of that equation is strictly concave on $[y^*, y_H^c]$, with a derivative that is strictly positive at y^* and at y_H^c (by (40)). Therefore, U_H^r is strictly increasing in y and maximized at y_H^c, where the constraint ceases to bind. ∎

Proof of Proposition 10. Given (41), the cap $\bar{Y}$ does not constrain low types from receiving their full symmetric-equilibrium allocation, $U_L^{SI} = w(y^*) + B\theta_L$. We look for a zero-profit, least-cost separating allocation, now satisfying

$$\pi(y_H) + B\theta_H + y_H b(y_H) \quad = \quad \overline{Y} + \zeta_H, \tag{C.36}$$

$$U_H - y_H \Delta\theta - \zeta_H \Delta\lambda \quad = \quad U_L^{SI}. \tag{C.37}$$

The first condition implies that

$$U_H = w(y_H) + B\theta_H - (1 - \lambda_H)\zeta_H = u(y_H) - y_H b(y_H) + \overline{Y} + \lambda_H \zeta_H, \tag{C.38}$$

and the second thus becomes

$$U_L^{SI} = u(y_H) - y_H b(y_H) + \overline{Y} + \lambda_L \zeta_H - y_H \Delta\theta. \tag{C.39}$$

Adding (C.36) to this last equation and substituting in U_L^* yields

$$\begin{aligned} w(y^*) + B\theta_L + \overline{Y} + \zeta_H &= w(y_H) + B\theta_H - y_H \Delta\theta + \overline{Y} + \lambda_L \zeta_H \iff \\ w(y^*) - w(y_H) &= (B - y_H)\Delta\theta - (1 - \lambda_L)\zeta_H \\ &= (B - y_H)\Delta\theta - (1 - \lambda_L)\left[\pi(y_H) + y_H b(y_H) + B\theta_H - \overline{Y}\right] \iff \\ w(y^*) &= w(y_H) + (B - y_H)\Delta\theta - (1 - \lambda_L)\left[A a(y_H) + B b(y_H) + B\theta_H - \overline{Y}\right], \end{aligned}$$

where the next-to-last equation uses (C.36) to substitute for ζ_H and the last one follows from the definition of π. Denoting $\varsigma(y_H)$ the right-hand side of this last equation, (10) implies that $\varsigma'(y_H) = w'(y_H) - (1 - \lambda_L)\left[w'(y_H) + y_H b'(y_H)\right] - \Delta\theta < 0$ for $y_H \geq y^*$, while (22) and (41) ensure that $\varsigma(y^*) > w(y^*) \geq \varsigma(y_H^c)$. Therefore, the equation has a unique solution $y_H^r \in (y^* y_H^c]$, which is easily seen to be increasing in $\bar{Y}$.

The next step consists once again in checking whether this allocation is interim efficient, and thus the equilibrium outcome. Consider therefore the following (relaxed) program:

$$\begin{array}{rcll}
(\mathcal{P}^r) & : & \max_{\{(0\le y_i\le \bar{y},\ 0\le\zeta_i,\ U_i)\}_{i=H,L}} \{U_H\}, \quad \text{subject to:} & \\
U_L & \ge & U_L^{SI} & (\nu) \quad (C.40)\\
U_L & \ge & U_H - y_H\Delta\theta - \zeta_H\Delta\theta & (\xi) \quad (C.41)\\
U_H & \le & u(y_H) - y_H b(y_H) + \overline{Y} + \lambda_H\zeta_H & (\kappa) \quad (C.42)\\
0 & \le & \sum_{i=H,L} [w(y_i) + \theta_i B - (1-\lambda_i)\zeta_i - U_i], & (\mu) \quad (C.43)
\end{array}$$

For the LCS allocation not to be interim efficient, the optimum must have $U_L > U_L^{SI}$, hence $\nu = 0$. Solving the first-order conditions in U_H and U_L then yields $\xi = q_L\mu$ and $\kappa = 1-\mu$. Furthermore, the first-order condition with respect to ζ_H is

$$\lambda_H(1-\mu) + q_L\mu\Delta\lambda - q_H\mu(1-\zeta_H) \le 0,$$

which is ruled out if q_H is small enough, in which case interim efficiency obtains.

Finally, let us examine how U_H varies with $\overline{Y}$. Equations (C.36) and (C.38) imply that

$$\begin{aligned}
dU_H &= \Delta\theta dy_H + \Delta\lambda d\zeta_H = w'(y_H)dy_H - (1-\lambda_H)d\zeta_H \\
&\Rightarrow -(1-\lambda_L)d\zeta_H = \left[\Delta\theta - w'(y_H)\right] dy_H
\end{aligned}$$

We saw earlier that y_H is increasing in $\bar{Y}$, and thus naturally ζ_H is increasing, as firms substitute toward the inefficient currency. Substituting for $d\zeta_H$ into the first equation, we have:

$$\frac{\partial U_H}{\partial \bar{Y}} = \left(\frac{w'(y_H)}{\Delta\theta} + \frac{1-\lambda_H}{\Delta\lambda}\right)\left(\frac{\Delta\theta\Delta\lambda}{1-\lambda_L}\right)\frac{\partial y_H}{\partial \bar{Y}}. \tag{C.44}$$

Under condition (C.35), the first term in brackets is positive, since $w'(y_H)/(\Delta\theta) > w'(y_H^c)/(\Delta\theta)$. Therefore, as $\bar{Y}$ is reduced, U_H decreases. Since low types' utility is unaffected and profits remain equal to zero, the result follows. ■

Proof of Proposition 11. Let us adopt the convention that z and y are *net* compensations. In particular, y is still the effective power of the incentive scheme. Profit on type $i = H, L$ under contract (y, z) is then

$$\Pi_i = Aa(y) + B[\theta_i + b(y)] - \frac{z + y[\theta_i + b(y)]}{1-\tau}, \tag{C.45}$$

while the expression for U_i is unchanged. Furthermore,

$$\begin{aligned}
U_i + (1-\tau)\Pi_i &= [(1-\tau)(Aa(y) + B[\theta_i + b(y)] - [C(a(y), b(y)) - va(y)] \\
&\equiv \widehat{w}(y) + (1-\tau)B\theta_i. \qquad (C.46)
\end{aligned}$$

Let $y^*(\tau) \le y^*$ be the bilaterally efficient power of incentives: $y^*(\tau) = \arg\max\{\widehat{w}(y)\}$. The LCS equilibrium has $y_L = y^*(\tau)$ and y_H given by

$$\widehat{w}(y^*(\tau)) - \widehat{w}(y_H) = \Delta\theta\left[(1-\tau)B - y_H\right]$$

Welfare W is equal to $q_H w(y_H) + q_L w(y_L) + B\overline{\theta}$, and so

$$\frac{dW}{d\tau}\mid_{\tau=0} = q_L w'(y_L)\frac{dy^*}{d\tau} + q_H w'(y_H)\frac{dy_H}{d\tau} = q_H w'(y_H)\frac{dy_H}{d\tau}. \tag{C.47}$$

Finally,

$$\begin{aligned} \widehat{w}'(y_H) &= w'(y_H) - \tau\frac{d}{dy_H}\left[Aa(y_H) + Bb(y_H)\right] \simeq w'(y_H) \Rightarrow \\ \frac{dW}{d\tau}\mid_{\tau=0} &\simeq -\frac{B\Delta\theta}{\Delta\theta - w'(y_H)} q_H w'(y_H) > 0. \blacksquare \end{aligned} \tag{C.48}$$

Lemma 8 *The first-best solution defined by (44) satisfies $y^{A*} < A$ and $y^{B*} < B$.*

Proof. The first-order conditions (44) take the form

$$\begin{aligned} (A - y^{A*})(\partial a/\partial y^{A*}) + (B - y^{B*})\left(\partial b/\partial y^A\right) &= ry^{A*}\sigma_A^2/2, \\ (A - y^{A*})(\partial a/\partial y^{B*}) + (B - y^{B*})\left(\partial b/\partial y^B\right) &= ry^{B*}\sigma_B^2/2, \end{aligned}$$

with all derivatives evaluated at (y^{A*}, y^{B*}). Let $D \equiv (\partial a/\partial y^A)\left(\partial b/\partial y^B\right) - (\partial a/\partial y^B)\left(\partial b/\partial y^A\right)$, which is easily seen to equal $1/[C_{aa}C_{bb} - (C_{ab})^2] > 0$ (this holds for any (y^A, y^B)). We then have

$$\begin{aligned} A - y^{A*} &= \frac{1}{D}\left[(\partial b/\partial y^B)\left(ry^{A*}\sigma_A^2/2\right) - (\partial b/\partial y^A)\left(ry^{B*}\sigma_B^2/2\right)\right] > 0, \\ B - y^{B*} &= \frac{1}{D}\left[(\partial a/\partial y^A)\left(ry^{B*}\sigma_B^2/2\right) - (\partial a/\partial y^B)\left(ry^{A*}\sigma_A^2/2\right)\right] > 0, \end{aligned}$$

hence the result. $\blacksquare$

Proof of Lemma 2. The LCS allocation is interim efficient if and only if it solves the relaxed program

$$\begin{aligned} &\max\{U_H\}, \text{ subject to} \\ &U_L \geq U_H - y_H^A\Delta\theta^A - y_H^B\Delta\theta^B, \\ &\sum_{i=H,L} q_i\left[w(y_i) + D_i - U_i\right] \geq 0, \\ &U_L \geq U_L^{SI}. \end{aligned}$$

The solution to this program must satisfy $y_L = y^*$. If the LCS allocation is not interim efficient, the solution must be such that $U_L > U_L^{SI}$, implying $\nu = 0$. Using the zero-profit condition, substituting U_L, using the incentive-compatibility condition and taking derivatives yields:

$$\frac{1}{\Delta\theta^A}\frac{\partial w(y_H)}{\partial y_H^A} = \frac{1}{\Delta\theta^B}\frac{\partial w(y_H)}{\partial y_H^B} = -\frac{q_L}{q_H}. \tag{C.49}$$

Letting σ denote the "subsidy" from the H- to the L-type, the above program can be rewritten as:

$$(\mathcal{P}^r): \quad \max\{U_H\}, \text{ subject to}$$
$$U_H \leq w(y_H) + D_H - \tfrac{q_L}{q_H}\sigma$$
$$U_L^{SI} + \sigma \geq U_H - y_H \cdot \Delta\theta$$
$$\sigma \geq 0$$

where $y_H \cdot \Delta\theta$ denotes the scalar product of $y_H \equiv \left(y_H^A, y_H^B\right)$ and $\Delta\theta \equiv \left(\Delta\theta^A, \Delta\theta^B\right)$. Note first that the first two constraints must both be binding. Indeed, denoting λ_i the Lagrange multiplier on the i-th constraint, the first-order conditions are $1-\lambda_1-\lambda_2 = 0$ for U_H, $\lambda_1 \nabla w(y_H)+\lambda_2\Delta\theta = 0$ for y_H and $\lambda_3-\lambda_1 q_L/q_H+\lambda_2 = 0$ for σ. The first two clearly exclude $\lambda_1 = 0$. If $\lambda_2 = 0$, then $y_H = y^*$ and $\lambda_3 > 0$, implying $\sigma = 0$; but then the second constraint becomes $w(y^*)+D_L = U_L^{SI} \geq w(y^*)+D_H-y^*\cdot\Delta\theta$, hence $0 \geq D_H - D_L - y^* \cdot \Delta\theta = (A - y_A^*)\Delta\theta^A + (B - y_B^*)\Delta\theta^B$, a contradiction of Lemma 8.

Next, eliminating σ from the binding constraints shows that y_H solves $\max\{w(y_H) + \ell\Delta\theta \cdot y_H\}$, where $\ell \equiv q_L/q_H \in (0, \infty)$ is the likelihood ratio. Consider any two such ratios ℓ and $\hat{\ell}$ and the corresponding optima y_H and $\widehat{y}_H$ for this last program; if $\hat{\ell} > \ell$, then

$$\begin{aligned} w(y_H) &\geq w(\hat{y}_H) + \ell\Delta\theta \cdot (\hat{y}_H - y_H), \\ w(\hat{y}_H) &\geq w(y_H) + \hat{\ell}\Delta\theta \cdot (y_H - \hat{y}_H). \end{aligned}$$

Adding up these inequalities yields $\Delta\theta \cdot (\hat{y}_H - y_H) \geq 0$, which in turn implies that $w(y_H) \geq w(\hat{y}_H)$. Observe now from (47) that the LCS allocation corresponds to an interior solution to $\max\{w(y_H) + \kappa^c\Delta\theta \cdot y_H\}$. Consider now any $\ell > \kappa^c$ and the corresponding solution y_H. We have $w(y_H) \leq w\left(y_H^c\right)$ and so

$$w(y_H) + D_H - \ell\sigma \leq w\left(y_H^c\right) + D_H = U_H^c,$$

with strict inequality if $\sigma > 0$. This last case is impossible, however, since type H's utility from the relaxed program cannot be lower than U_H^c. Therefore, $\sigma = 0$ and $y_H = y_H^c$: the LCS allocation is interim efficient. Conversely, let $\ell < \kappa^c$; we have

$$\frac{\partial}{\partial y_H}\left[w(y_H) + \ell\Delta\theta \cdot y_H\right]_{y_H = y_H^c} = (l - \kappa^c)\,\Delta\theta,$$

with $\Delta\theta^A \geq 0$ and $\Delta\theta^B > 0$. Since y_H maximizes the expression in brackets, it must be that $y_H^A \leq y_H^{Ac}$ and $y_H^B < y_H^{Ac}$, hence $\Delta\theta \cdot (y_H^c - y_H) > 0$. By the same properties shown above, it follows that $w(y_H) > w(y_H^c)$. If $\sigma = 0$, the two binding constraints in $(\mathcal{P}^r)$ then imply that

$$\begin{aligned} U_H &= U_L^{SI} + y_H \cdot \Delta\theta < U_L^{SI} + y_H^c \cdot \Delta\theta = U_H^c, \\ U_H &= w(y_H) + D_H > w(y_H^c) + D_H = U_H^c, \end{aligned}$$

another contradiction. Therefore σ must be positive after all, and interim efficiency fails. ■

Appendix D: Additional Proofs

D.1 Bounds on $\bar{U}$ ensuring non-negative equilibrium wages.

We make explicit here the restrictions on $\bar{U}$ ensuring that: (i) $z_H(t)$ be non negative, for all t; (ii) [34]firms want to keep low types one board. We then provide sufficient conditions for all of them to hold jointly. In Region III, $z_H(t) = \bar{U} + \hat{y}_L(t)\Delta\theta - u(y^*) - \theta_H y^*$ is decreasing, so it suffices that $\lim_{t\to+\infty} z_H(t) = z_H^m \geq 0$. In Regions I and II, $z_H(t) = U_L(t) - \theta_L \hat{y}_H(t) - u(\hat{y}_H(t))$; its variations with t are ambiguous, but since $U(t)$ is (weakly) declining toward $\bar{U}$ and $\hat{y}_H(t)$ strictly decreasing from $\hat{y}_H^c(0) = y_H^c$, it is bounded below by $\bar{U} - \theta_L y_H^c - u(y_H^c)$. Combining this with condition (16), we therefore require:

$$\bar{U}_{\min} \equiv \max\left\{u(y^*) + \theta_H y^* - y_L^m \Delta\theta,\ u(y_H^c) + \theta_L y_H^c\right\} \leq \bar{U} \leq w(y_L^m) + \theta_L B - (q_H/q_L) y_L^m \Delta\theta \equiv \bar{U}_{\max}. \tag{D.1}$$

This defines a nonempty interval for $\bar{U}$ as long as $\bar{U}_{\min} < \bar{U}_{\max}$, which can be insured in at least two ways. First, for q_L close enough to 1 (thus also satisfying the requirement of (30)) y_L^m is close to y^*, so $\bar{U}_{\min} \approx\ u(y_H^c) + \theta_L y_H^c < w(y_H^c) + \theta_L y_H^c < w(y^*) + \theta_L B \approx \bar{U}_{\max}$, ensuring the result.

Alternatively, one can slightly modify firms' technology so that the revenue generated by each worker of type θ becomes instead $Aa + B(\theta + b) + \bar{d}$, where $\bar{d}$ is a constant reflecting some other "basic" activity, performed at a fixed (e.g., perfectly monitored) level by all employees, and for which their compensation is therefore part of the fixed wage z. This augments total surplus $w(y)$ by the same amount $\bar{d}$, which can be made large enough to ensure that $\bar{U}_{\min} < \bar{U}_{\max}$. ■

D.2 General optimization program

Let $\widehat{\mathcal{C}} \equiv (\hat{U}_H, \hat{U}_L, \hat{y}_H, \hat{y}_L)$ denote the (presumptive) symmetric-equilibrium strategies and payoffs, given in Proposition 4, and played by the other firm. For all $u \in \mathbb{R}$, let $\mathcal{X}(u) \equiv \min\left\{\max\left\{u, 0\right\}, 2t\right\}.$ The firm's general problem is to choose (U_H, U_L, y_H, y_L) to solve the program:

$$\max \Big\{ q_H \mathcal{X}(U_H + t - \hat{U}_H)[w(y_H) + \theta_H B - U_H \\ + q_L\ \mathcal{X}(U_L + t - \hat{U}_L) \mathbf{1}_{\left\{U_L \geq \bar{U}\right\}} [w(y_L) + \theta_L B - U_L] \Big\} \tag{D.2}$$

subject to:

$$U_H \ \geq\ U_L + y_L \Delta\theta \tag{D.3}$$

$$U_L \ \geq\ U_H - y_H \Delta\theta \tag{D.4}$$

Note that the objective function (D.2) is not everywhere differentiable, nor (as we shall see), is it globally concave. Note also that if either $U_L \leq \hat{U}_L - t$ or $U_L < \bar{U}$, the firm employs zero (measure of) low types, in which case it clearly must sell to a positive measure of H agents, requiring $U_H > \max\{\hat{U}_H - t, \bar{U}\}$. We first rule out such "exclusion" of low-skill workers, and likewise for high-skill ones. We then show that is also not optimal to "corner" the market on either type.

[34]Recall that $z_L(t) \geq z_H(t)$ everywhere by incentive compatibility. As to the bonus rates $yi(t)$, they all are bounded below by y_L^m, which is positive since we have assumed that $w'(0) > (q_H/q_L)\Delta\theta$.

D.2.1 No exclusion and no cornering

Lemma 9 *There exists $\bar{q}_L \in [q_L^*, 1)$, independent of t, such that, for all $q_L \geq \bar{q}_L$, it is strictly suboptimal not to employ a positive measure of L-type agents. In particular, $U_L \geq \bar{U}$.*

Proof. Selling only to H agents under some contract (y_H, U_H) is less profitable than sticking to the symmetric strategy $(\hat{y}_H, \hat{U}_H)$ if

$$\begin{aligned} q_H \pi_H &\equiv q_H \chi(U_H - \hat{U}_H + t)\left[w(y_H) + B\theta_H - U_H\right] \\ &\leq q_H t[w(\hat{y}_H) + B\theta_H - \hat{U}_H] + q_L t[w(\hat{y}_L) + B\theta_L - \hat{U}_L] \equiv q_H \hat{\pi}_H + q_L \hat{\pi}_L \equiv \hat{\pi}. \quad \text{(D.5)} \end{aligned}$$

For any $t > 0$, $\hat{\pi}_L > 0$, so the inequality is satisfied for q_H low enough, or equivalently q_L/q_H large enough. To ensure a lower bound independent of t, however, the ratio $(\pi_H - \hat{\pi}_H)/\hat{\pi}_L$ must remain bounded above as t tends to zero, even though $\lim_{t \to 0} \hat{\pi}_L = 0$. We will in fact show a stronger property, namely that $\pi_H(t) < \hat{\pi}_H(t)$ for t small enough.

Observe first that to exclude the L types, it must be that $U_L \leq \max\{\bar{U}, \hat{U}_L - t\}$. For all $t < t_1$ we have $\hat{U}_L > \bar{U}$, so for small t the relevant constraint is $U_L \leq \hat{U}_L - t$. The firm thus solves:

$$\begin{aligned} &\max \left\{ \chi(U_H - \hat{U}_H + t)\left[w(y_H) + B\theta_H - U_H\right] \right\}, \quad \text{subject to:} \\ &U_H \geq U_L + y_L \Delta\theta \qquad (\mu_H) \\ &U_L \geq U_H - y_H \Delta\theta \qquad (\mu_L) \\ &U_L \leq \hat{U}_L - t \qquad (\varphi) \\ &y_L \geq 0 \qquad (\psi). \end{aligned}$$

To have a positive share of the H types it must be that $U_H - \hat{U}_H > -t > U_L - \hat{U}_L$, therefore $U_H - U_L > \hat{U}_H - \hat{U}_L = \hat{y}_H \Delta\theta$, implying $y_H > \hat{y}_H$. Consider now the first-order conditions:

$$\begin{aligned} &-1 \leq \mu_L - \mu_H \leq w(y_H) + B\theta_H - 2U_H + \hat{U}_H - t, \\ &\text{with equality for } U_H - \hat{U}_H > t \text{ and } U_H - \hat{U}_H < t, \text{ respectively;} \\ &-\mu_H + \mu_L - \varphi = 0, \\ &(U_H - \hat{U}_H + t)w'(y_H) + \mu_L \Delta\theta = 0, \\ &\psi - \mu_H \Delta\theta = 0. \end{aligned}$$

If $\mu_L = 0$, the third condition implies that $y_H = y^* \leq \hat{y}_H$, a contradiction. Therefore $\mu_L > 0$, so that $U_H - U_L = y_H \Delta\theta$, with $\hat{y}_H < y_H$. Next, it cannot be that $\psi > 0$, otherwise $y_L = 0$ and $U_H - U_L = y_L \Delta\theta$ so $y_H = y_L = 0$, another contradiction. Hence $\mu_H = 0$, so $\varphi = \mu_L > 0, U_L = \hat{U}_L - t$ and therefore $U_H - \hat{U}_H + t = (y_H - \hat{y}_t)\Delta\theta$, which furthermore cannot exceed $2t$, since $-1 < \mu_L - \mu_H$. Next, eliminating μ_L,

$$w(y_H) + B\theta_H - 2U_H + \hat{U}_H - t + (U_H - \hat{U}_H + t)w'(y_H)/\Delta\theta \geq 0, \tag{D.6}$$

with equality for $U_H - \hat{U}_H < t$.

We also have, from (36) and (38)-(39) with $\hat{y}_L = y^*$, a similar condition (with equality) for $\hat{y}_H$:

$$w(\hat{y}_H) + B\theta_H - \hat{U}_H - t + tw'(\hat{y}_H)/\Delta\theta = 0. \tag{D.7}$$

Subtracting and replacing $U_H - \hat{U}_H + t$ by $(y_H - \hat{y}_t)\Delta\theta$ yields:

$$\begin{aligned}\Upsilon(y_H; \hat{y}_H, t) \equiv w(y_H) - w(\hat{y}_H) - 2\left[(y_H - \hat{y}_H)\Delta\theta - t\right] \\ +(y_H - \hat{y}_H)w'(y_H) - tw'(\hat{y}_H)/\Delta\theta \geq 0,\end{aligned} \tag{D.8}$$

with equality for $U_H - \hat{U}_H < t$. If cannot be that $U_H - \hat{U}_H = t$, moreover, otherwise $(y_H - \hat{y}_t)\Delta\theta = 2t$ and $\Upsilon(y_H; \hat{y}_H, t) = w(y_H) - w(\hat{y}_H) - 2t + tw'(\hat{y}_H)/\Delta\theta < 0$, a contradiction. Therefore (D.8) is an equality, and since $\partial\Upsilon/\partial y_H = 2w'(y_H) - 2\Delta\theta + y_H w''(y_H) < 0$, it uniquely defines y_H as a function $y_H = Y(\hat{y}_H, t)$. Taken now as a function of t, $y_H(t) = Y(\hat{y}_H(t), t)$ tends to $Y(\hat{y}_H(0), 0) = \hat{y}_H(0) = y_H^c$, as can be seen from taking limits in (D.8) as an equality. A Taylor expansion of $\Upsilon(y_H(t); \hat{y}_H(t), t) = 0$ then yields

$$\begin{aligned} 2\left[\Delta\theta - w'(y_H^c)\right](y_H(t) - \hat{y}_H(t)) &= t\left[2 - w'(y_H^c)/\Delta\theta\right] + \mathcal{O}(t^2) \Rightarrow \\ y_H(t) - \hat{y}_H(t) &= \omega t + \mathcal{O}(t^2), \end{aligned} \tag{D.9}$$

where $\omega \equiv \left[2 - w'(y_H^c)/\Delta\theta\right] / \left[2\Delta\theta - 2w'(y_H^c)\right] \in (0, 1)$. Turning now to the associated profit margins, we have from (D.7) and (D.6) (now known to be an equality) respectively,

$$\begin{aligned} w(\hat{y}_H) + B\theta_H - \hat{U}_H &= t[1 - w'(\hat{y}_H)/\Delta\theta], \\ w(y_H) + B\theta_H - U_H &= (U_H - \hat{U}_H + t)[1 - w'(y_H)/\Delta\theta]. \end{aligned}$$

Consequently, as $t \to 0$,

$$\frac{\pi_H(t)}{\hat{\pi}_H(t)} = \frac{(U_H - \hat{U}_H + t)^2}{t^2} \frac{1 - w'(y_H(t))/\Delta\theta}{1 - w'(\hat{y}_H(t))/\Delta\theta} \to (\omega\Delta\theta)^2 < 1,$$

which concludes the proof. ■

We now rule out excluding high-skill workers.

Lemma 10 *It is always strictly suboptimal not to employ a positive measure of H-type agents.*

Proof. If a firm, say Firm 0, employs no H agent it must sell to a positive measure of L agents and reap strictly positive profits from their contract (y_L, U_L). Furthermore, the optimal level of y_L is clearly y^*. Thus, it must be that $\bar{U} \leq U_L$ and $\hat{U}_L - t < U_L < w(y^*) + B\theta_L$.

In Region III, let the firm deviate and offer the single contract (y_L, U_L). By taking it, an agent of type H gets $\tilde{U}_H = U_L + y^*\Delta\theta > \hat{U}_L - t + y^*\Delta\theta \geq \hat{U}_H - t$, so it is preferred by a positive measure of them to going to work for Firm 1, as well as to the outside option ($\tilde{U}_H > \bar{U}$). Each of these workers then generates profits $w(y^*) + B\theta_H - \tilde{U}_H = w(y^*) + B\theta_L - U_L + (B - y^*)\Delta\theta > 0$. Therefore, a contract excluding H workers could not in fact have been optimal.

In Regions I and II, we will show that there always exists a contract $(\tilde{y}_H, \tilde{U}_H)$ that can be offered alongside with (y_L, U_L) so as to attract a positive measure of H types, not be strictly preferred by any L type, and generate positive profits. Note first that if $U_L \geq \hat{U}_L$, we can simply choose $(\tilde{y}_H, \tilde{U}_H) = (\hat{y}_H, \hat{U}_H)$, that is, the same contract as offered by Firm 1. Indeed, since $U_L \geq \hat{U}_L \geq \hat{U}_H - \hat{y}_H \Delta\theta = \tilde{U}_H - \tilde{y}_H \Delta\theta$, the L types employed at Firm 0 (weakly) prefer their original contract, (y_L, U_L). For the H types, clearly $\tilde{U}_H = \hat{U}_H > \bar{U}$ and getting it from Firm 0 is preferable to getting it from Firm 1 for all such agents located at $x < 1/2$. Such a deviation is thus strictly profitable.

Suppose from now on that $U_L < \hat{U}_L$ and consider the contract $(\tilde{y}_H, \tilde{U}_H) \equiv (\hat{y}_H, U_L + \hat{y}_H \Delta\theta)$. The L types have no reason to switch (they are indifferent), while for the H types we have $\tilde{U}_H = U_L + \hat{y}_H \Delta\theta > \hat{U}_L + \hat{y}_H \Delta\theta - t = \hat{U}_H - t$, so a positive measure of them prefer this new offer to what they could get at Firm 1. Furthermore, since $\tilde{U}_H \geq U_L + y^* \Delta\theta$, they also prefer it to the L types' contract at Firm 0. The firm can thus offer the incentive-compatible menu $\{(y_L, U_L), (\tilde{y}_H, \tilde{U}_H)\}$ and attract a positive measure of H agents, on which it makes unit profit

$$
\begin{aligned}
w(\hat{y}_H) + B\theta_H - \tilde{U}_H &= w(\hat{y}_H) + B\theta_H - \hat{y}_H \Delta\theta - U_L \\
&> w(\hat{y}_H) + B\theta_H - \hat{y}_H \Delta\theta - \hat{U}_L = w(\hat{y}_H) + B\theta_H - \hat{U}_H > 0.
\end{aligned}
$$

The deviation is therefore profitable, which concludes the proof. ■

D.2.3 A key property

By Lemmas 9 and 10, at an optimum it must be that $X_H \equiv \mathcal{X}(U_H + t - \hat{U}_H) > 0$ and $X_L \equiv \mathcal{X}(U_L + t - \hat{U}_L)\mathbf{1}_{\{U_L \geq \bar{U}\}} > 0$. This, in turn, implies:

Lemma 11 *At any optimum, it must be that either:*
(i) $y_L = y^* \leq y_H$ *and* $U_H - U_L = y_H \Delta\theta$, *with multiplier* $\mu_H = 0$ *on (D.3), or*
(ii) $y_L \leq y^* = y_H$ *and* $U_H - U_L = y_L \Delta\theta$, *with multiplier* $\mu_L = 0$ *on (D.4).*

Proof. Consider the sub-problem of maximizing (D.2) over (y_H, y_L), while keeping (U_H, U_L) and therefore $(X_H > 0,\ X_L > 0)$ fixed. This is a differentiable and concave problem, for which the first-order conditions are:

$$0 = q_H X_H w'(y_H) + \mu_L \Delta\theta, \tag{D.10}$$

$$0 = q_L X_L w'(y_L) - \mu_H \Delta\theta + \psi. \tag{D.11}$$

Once again it cannot be that $\mu_H > 0$ and $\mu_L > 0$, otherwise (D.3)-(D.4) and (D.10) imply that $y_L = y_H > y^*$ and so $\psi = 0$, yielding a contradiction in (D.11). Suppose first that $\mu_H = 0$, implying that $\psi = 0$ and $y_L = y^*$. If (D.4) were not binding, we would have $\mu_L = 0$, hence $y_H = y^* = y_L$ and $U_L > U_H - y_H \Delta\theta = U_H - y_L \Delta\theta \geq U_L$, a contradiction. Thus it must be that $y_H \Delta\theta = U_H - U_L \geq y_L \Delta\theta = y^* \Delta\theta$, which corresponds to case (i). If $\mu_H > 0$, then (D.3) is binding and μ_L must equal 0, hence $y_H = y^*$. Furthermore, $y_L \Delta\theta = U_H - U_L \leq y_H \Delta\theta = y^* \Delta\theta$, which corresponds to case (ii). ■

We now show that it is also not optimal to (strictly) "corner" the market for either type, offering more utility than needed to attract the most distant employees away from the other firm.

Lemma 12 *At an optimum, $X_H \equiv U_H + t - \hat{U}_H$ and $X_L \equiv U_L + t - \hat{U}_L$ must both lie in $(0, 2t]$.*

Proof. The fact that $X_H > 0$ and $X_L > 0$ was established previously. Suppose first that $\min\{U_H + t - \hat{U}_H, U_L + t - \hat{U}_L\} > 2t$. Note that this implies $U_L > \hat{U}_L + t > \bar{U}$. The firm can then reduce both U_H and U_L slightly while keeping the full market of both types, $X_H = X_L = 1$ and not violating any constraint; this increases profits, a contradiction.

Suppose next that $U_H + t - \hat{U}_H \leq 2t < U_L + t - \hat{U}_L$, which again implies $U_L > \bar{U}$; furthermore, one must also have $U_H - U_L \leq \hat{U}_H - \hat{U}_L$. The chosen allocation must thus solve

$$\max \left\{ q_H \chi(U_H + t - \hat{U}_H)[w(y_H) + \theta_H B - U_H] + q_L(2t)[w(y_L) + \theta_L B - U_L] \right\},$$

subject again to (D.3)-(D.4), plus the participation constraint $U_L \geq \bar{U}$, which in this particular case is not binding. Maximizing over U_L thus yields the first-order condition $-2tq_L - \mu_H + \mu_L = 0$, which must hold in addition to (D.10)-(D.11) with $X_L = 1$. Clearly, it cannot be that $\mu_L = 0$. Therefore, $\mu_H = 0 < \mu_L = 2tq_L$, implying that (D.10) becomes $q_H(X_H/2t)w'(y_H) + q_L\Delta\theta = 0$. Furthermore, $y_H\Delta\theta = U_H - U_L \leq \hat{U}_H - \hat{U}_L \leq y_H^c\Delta\theta$, so $y_H \leq y_H^c$. But then the interim-efficiency condition (23) implies that $q_H w'(y_H) + q_L\Delta\theta > 0$, a contradiction since $X_H \leq 2t$.

Suppose now that $U_L + t - \hat{U}_L \leq 2t < U_H + t - \hat{U}_H$. The allocation must be a solution to

$$\max \left\{ q_H(2t)[w(y_H) + \theta_H B - U_H] + q_L \chi(U_L + t - \hat{U}_L)[w(y_L) + \theta_L B - U_L] \right\},$$

subject to (D.3)-(D.4) and the constraint $U_L \geq \bar{U}$, with associated multiplier $\nu \geq 0$. Maximizing over U_H thus yields the first-order condition $-2tq_H + \mu_H - \mu_L = 0$.This precludes $\mu_H = 0$, so $\mu_L = 0 < \mu_H = 2tq_H$, $y_H = y^*$ and $q_L X_L w'(y_L) = 2tq_H\Delta\theta - \psi \equiv 2tq_L w'(y_L^m) - \psi$. If $\psi > 0$ then $y_L = 0 < y_L^m$, and if $\psi = 0$ then $(X_L/2t)\, w'(y_L) = w'(y_L^m)$ so $y_L < y_L^m$, as $X_L \leq 2t$. But we also have $y_L\Delta\theta = U_H - U_L > \hat{U}_H - \hat{U}_L > y_L^m\Delta\theta$, a contradiction. ■

D.2.4 Proof of global optimality

The objective function in (D.14) is not globally concave, as can be seen computing the Hessian. The proof of global optimality will therefore require several steps. First, we will show that for any $\mathcal{C} = (U_H, U_L, y_H, y_L)$ to be an optimum, it must lie in either the following subspaces:

$$S_H \equiv \{(U_H, U_L, y_H, y_L)\, | y_L = y^* \leq y_H \leq y_H^c \text{ and } U_H - U_L = y_H\Delta\theta\}, \tag{D.12}$$

$$S_L \equiv \{(U_H, U_L, y_H, y_L)\, | y_H = y^* \geq y_L \geq y_L^m \text{ and } U_H - U_L = y_L\Delta\theta\}. \tag{D.13}$$

We will then show that the program is strictly concave on S_H and on S_L separately, which implies that $\widehat{\mathcal{C}} = (\hat{U}_H, \hat{U}_L, \hat{y}_H, \hat{y}_L)$ achieves a maximum over all feasible allocations in the subspace to which it belongs, namely S_H for $t \leq t_2$ (Regions I and II), or S_L for $t \geq t_2$ (Region III). Finally, we will show that the global optimum can never lie in the other subspace than the one to which $\widehat{\mathcal{C}}$ belongs, concluding the proof.

Lemma 13 *A global optimum $C = (U_H, U_L, y_H, y_L)$ must lie in S_H or in S_L.*

Proof. Let S'_H be denote the superset of S_H obtained by omitting the inequality $y_H \leq y_H^c$ from (D.12), and similarly let S'_L denote the superset of S_L obtained by omitting the inequality $y_L \geq y_L^m$ from (D.13). By Lemma 11, an optimum must belong to S'_H or S'_L. Furthermore, given no exclusion nor strict cornering (Lemmas 9, 10 and 12), solving (D.2)-(D.4) is equivalent to solving the smooth program

$$\max q_H(U_H + t - \hat{U}_H)[w(y_H) + \theta_H B - U_H] + q_L(U_L + t - \hat{U}_L)[w(y_L) + \theta_L B - U_L] \text{ ,(D.14)}$$

subject to:

$$\begin{array}{ll} X_H \equiv U_H + t - \hat{U}_H \leq 2t & (\tau_H) \\ X_L \equiv U_L + t - \hat{U}_L \leq 2t & (\tau_L) \\ U_H \geq U_L + y_L\Delta\theta & (\mu_H) \\ U_L \geq U_H - y_H\Delta\theta & (\mu_L) \\ U_L \geq \bar{U} & (\nu) \\ y_L \geq 0 & (\psi). \end{array}$$

The first-order conditions are:

$$\begin{aligned} q_H\left[w(y_H) + B\theta_H - 2U_H + \hat{U}_H - t\right] + \mu_H - \mu_L - \tau_H &= 0, & \text{(D.15)} \\ q_L\left[w(y_L) + B\theta_L - 2U_L + \hat{U}_L - t\right] + \mu_L - \mu_H + \nu - \tau_L &= 0, & \text{(D.16)} \\ q_H\left(U_H - \hat{U}_H + t\right) w'(y_H) + \mu_L\Delta\theta &= 0, & \text{(D.17)} \\ q_L\left(U_L - \hat{U}_L + t\right) w'(y_L) - \mu_H\Delta\theta + \psi &= 0 & \text{(D.18)} \end{aligned}$$

and we also know that $X_H > 0$ and $X_L > 0$ at an optimum.

Case A. Consider first $C \in S'_H$. We have $y_L = y^*$ (so $\psi = 0$) and $\mu_H = 0$, so eliminating μ_L :

$$\begin{aligned} w(y_H) + B\theta_H - 2U_H + \hat{U}_H - t + (U_H - \hat{U}_H + t)\frac{w'(y_H)}{\Delta\theta} - \frac{\tau_H}{q_H} &= 0, & \text{(D.19)} \\ w(y^*) + B\theta_L - 2U_L + \hat{U}_L - t - \frac{q_H}{q_L}(U_H - \hat{U}_H + t)\frac{w'(y_H)}{\Delta\theta} + \frac{\nu}{q_L} - \frac{\tau_L}{q_L} &= 0. & \text{(D.20)} \end{aligned}$$

Subtracting and using $U_H - U_L = y_H\Delta\theta$ and $\hat{U}_H - \hat{U}_L = \hat{y}\Delta\theta$ (with $\hat{y} = \hat{y}_H$ in Regions I and II and $\hat{y} = \hat{y}_L$ in Region III) yields

$$\begin{aligned} w(y_H) - w(y^*) + (B - y_H)\Delta\theta &= (y_H - \hat{y})\Delta\theta + (U_H - \hat{U}_H + t)\left[1 - \frac{w'(y_H)}{\Delta\theta}\right] \\ &\quad -(U_H - \hat{U}_H + t)\left[1 + \frac{q_H}{q_L}\frac{w'(y_H)}{\Delta\theta}\right] + \frac{\nu}{q_L} + \frac{\tau_H}{q_H} - \frac{\tau_L}{q_L}. \end{aligned}$$

Next, subtracting $w(y_H^c) - w(y^*) + (B - y_H^c)\Delta\theta = 0$, we have

$$\begin{aligned} & w(y_H) - w(y_H^c) - (y_H - y_H^c)\,\Delta\theta - (y_H - \hat{y})\Delta\theta \\ = \; & (U_H - \hat{U}_H + t)\left(-1 - \frac{q_H}{q_L}\right)\frac{w'(y_H)}{\Delta\theta} + \frac{\nu}{q_L} + \frac{\tau_H}{q_H} - \frac{\tau_L}{q_L}, \end{aligned} \tag{D.21}$$

or:

$$w(y_H) - w(y_H^c) - (2y_H - y_H^c - \hat{y})\,\Delta\theta = (U_H - \hat{U}_H + t)\frac{-w'(y_H)}{q_L\Delta\theta} + \frac{\nu}{q_L} + \frac{\tau_H}{q_H} - \frac{\tau_L}{q_L}.$$

If $y_H > y_H^c \geq \hat{y}$ the left-hand side is negative, while the right-hand side is positive, since $U_H - U_L > \hat{U}_H - \hat{U}_L$ implies that $U_L - \hat{U}_L < U_H - \hat{U}_H \leq 2t$, so $\tau_L = 0$. Hence, a contradiction, from which we conclude that $y_H \leq y_H^c$, so that $C \in S_H$.

Case B. Consider now $C \in S_L'$. We have $y_H = y^*$ and $\mu_L = 0$, so eliminating μ_H :

$$w(y^*) + B\theta_H - 2U_H + \hat{U}_H - t + \frac{q_L}{q_H}(U_L - \hat{U}_L + t)\frac{w'(y_L)}{\Delta\theta} + \frac{\psi}{q_H\Delta\theta} - \frac{\tau_H}{q_H} = 0, \tag{D.22}$$

$$w(y_L) + B\theta_L - 2U_L + \hat{U}_L - t - (U_L - \hat{U}_L + t)\frac{w'(y_L)}{\Delta\theta} + \frac{\nu}{q_L} - \frac{\psi}{q_L\Delta\theta} - \frac{\tau_L}{q_L} = 0. \tag{D.23}$$

If $y_L < y_L^m$ then $U_H - U_L = y_L\Delta\theta < \hat{y}\Delta\theta = \hat{U}_H - \hat{U}_L$ so $U_H - \hat{U}_H < U_L - \hat{U}_L \leq 2t$, hence $\tau_H = 0$.

Suppose first that $U_L > \bar{U}$; then $\nu = 0$ and from the two above equations we have

$$w(y^*) + B\theta_H - 2U_H + \hat{U}_H - t < 0 < w(y_L) + B\theta_L - 2U_L + \hat{U}_L - t.$$

But:

$$\begin{aligned} & w(y^*) + B\theta_H - 2U_H + \hat{U}_H - \left[w(y_L) + B\theta_L - 2U_L + \hat{U}_L\right] \\ = \; & w(y^*) - w(y_L) + (B - y_L)\,\Delta\theta + (\hat{y} - y_L)\Delta\theta > 0, \end{aligned}$$

a contradiction. Therefore, $U_L = \bar{U}$. Next, for $y_L < y_L^m$ we have $w'(y_L) > w'(y_L^m) = (q_H/q_L)\,\Delta\theta$, hence, by (D.22):

$$\begin{aligned} -\psi/q_H\Delta\theta \; & > \; \theta w(y^*) + B\theta_H - 2U_H + \hat{U}_H - t + U_L - \hat{U}_L + t \\ & = \; w(y^*) + B\theta_L - \bar{U} + (B - y_L)\Delta\theta - \left[U_H - U_L - (\hat{U}_H - \hat{U}_L\right] \\ & = \; w(y^*) + B\theta_L - \bar{U} + (B - y_L)\Delta\theta + (\hat{y} - y_L)\,\Delta\theta. \end{aligned}$$

This last expression is strictly positive, however, since $y_L < y_L^m \leq \hat{y} < B$, where $\hat{y} = \hat{y}_H$ when t is in Region I or II and $\hat{y} = \hat{y}_L$ when t is in Region III. Hence another contradiction, from which we conclude that $y_L \geq y_L^m$, so that $C \in S_L$. ■

Lemma 14 *The objective function in (D.14) is strictly concave over S_H and over S_L.*

In passing, note that this result implies that the symmetric solution $\widehat{\mathcal{C}} \equiv (\hat{U}_H, \hat{U}_L, \hat{y}_H, \hat{y}_L)$ always satisfies the *local* second-order conditions for a maximum of the program (D.14).[35]

Proof. First, over S_H, the objective function becomes

$$\begin{aligned}\phi(U_H, y_H) \equiv\ & q_H(U_H - \hat{U}_H + t)\left[w(y_H) + \theta_H B - U_H\right] \\ & + q_L(U_H - y_H\Delta\theta - \hat{U}_L + t)\left[w(y^*) + \theta_L B - U_H + y_H\Delta\theta\right], \end{aligned} \quad (D.24)$$

for which the Hessian is

$$H(\phi) = \begin{bmatrix} -2 & q_H w'(y_H) + 2q_L\Delta\theta \\ q_H w'(y_H) + 2q_L\Delta\theta & q_H(U_H - \hat{U}_H + t)w''(y_H) - 2q_L\Delta\theta^2 \end{bmatrix}$$

and its determinant equals

$$\begin{aligned} & -q_H^2 w'(y_H)^2 - 4q_H q_L w'(y_H)\Delta\theta - 4q_L^2\Delta\theta^2 + 4q_L\Delta\theta^2 - 2q_H w''(y_H)\left(U_H - \hat{U}_H + t\right) \\ = & -q_H w'(y_H)\left[q_H w'(y_H) + 4q_L\Delta\theta\right] + 4q_H q_L\Delta\theta^2 - 2q_H w''(y_H)\left(U_H - \hat{U}_H + t\right), \end{aligned}$$

which is positive since $y_H \leq y_H^c$ implies that $q_H w'(y_H) + 4q_L\Delta\theta \geq q_H w'(y_H^c) + 4q_L\Delta\theta > 0$, by (23).

Next, over S_L, the objective function becomes

$$\begin{aligned}\phi(U_L, y_L) \equiv\ & q_H(U_L + y_L\Delta\theta - \hat{U}_H + t)[w(y^*) + \theta_H B - U_L - y_L\Delta\theta\ ] \\ & + q_L(U_L + t - \hat{U}_L)[w(y_L) + \theta_L B - U_L], \end{aligned} \quad (D.25)$$

for which the Hessian is

$$H(\phi) = \begin{bmatrix} -2 & q_L w'(y_L) - 2q_H\Delta\theta \\ q_L w'(y_L) - 2q_H\Delta\theta & q_L(U_L - \hat{U}_L + t)w''(y_L) - 2q_H\Delta\theta^2 \end{bmatrix}$$

and its determinant equals:

$$\begin{aligned} & -q_L^2 w'(y_L)^2 + 4q_H q_L w'(y_L)\Delta\theta - 4q_H^2\Delta\theta^2 + 4q_H\Delta\theta^2 - 2q_L w''(y_L)(U_L - \hat{U}_L + t) \\ = \ & q_L w'(y_L)\left[-q_L w'(y_L) + 4q_H\Delta\theta\right] + 4q_H q_L\Delta\theta^2 - 2q_L w''(y_L)(U_L - \hat{U}_L + t), \end{aligned}$$

which is positive since $y_L \geq y_L^m$ implies $q_L w'(y_L) \leq q_L w'(y_L^m) < 4q_H\Delta\theta$, by (10). ■

Proposition 20 *The unique global optimum to (D.2)-(D.4) is the allocation* $\widehat{\mathcal{C}} \equiv (\hat{U}_H, \hat{U}_L, \hat{y}_H, \hat{y}_L)$ *characterized in Proposition 4, which is therefore an equilibrium (the unique symmetric one) of the game between the two firms.*

Proof. By Lemmas 9 and 10, the global solution $\mathcal{C} = (U_H, U_L, y_H, y_L)$ to (D.2)-(D.4) is also the global solution to (D.14) and satisfies the associated first-order condition (D.15)-(D.18), with

[35] This can also be shown directly, by computing the appropriate bordered Hessians, given each of the constraints binding in Regions I, II and III respectively. The proof is available upon request.

$X_H \equiv U_H - \hat{U}_H + t$ and $X_L \equiv U_H - \hat{U}_H + t$ both in $(0, 2t]$. By Proposition4, the symmetric allocation $\widehat{\mathcal{C}} \equiv (\hat{U}_H, \hat{U}_L, \hat{y}_H, \hat{y}_L)$ solves these first-order conditions (with $\hat{X}_H = \hat{X}_L = t$) and is such that $\widehat{\mathcal{C}} \in S_H$ when t is in Regions I and II, while $\widehat{\mathcal{C}} \in S_L$ when t is in Region III. Furthermore, by Lemma 14, the objective function is strictly concave over each of these subspaces, so in each case $\widehat{\mathcal{C}}$ maximizes the program over the subspace to which it belongs. By Lemma 14, moreover, the global optimum $\mathcal{C}$ must also belong to S_H or S_L. Two cases therefore remain to consider .

Case A: t lies in Region I or II, so that $\widehat{\mathcal{C}} \in S_H$. If $\mathcal{C} \in S_H$ as well, they must coincide. If $\mathcal{C} \in S_L$ then $y_H = y^*$, $\mu_L = 0$ and

$$U_H - U_L = y_L \Delta\theta \ \leq \hat{y}_H \Delta\theta = \hat{U}_H - \hat{U}_L. \tag{D.26}$$

• *Subcase A1.* If the inequality is strict then

$$U_H - \hat{U}_H \ < U_L - \hat{U}_L. \tag{D.27}$$

Note that this requires $\tau_H = 0$, otherwise $t = U_H - \hat{U}_H \ < U_L - \hat{U}_L \leq t$, a contradiction. Next, subtracting from (D.15) its counterpart for $\widehat{\mathcal{C}}$, and likewise for (D.16), we have:

$$\begin{aligned} q_H \left[w(y^*) + B\theta_H - 2U_H + \hat{U}_H - t \right] + \mu_H &= q_H \left[w(\hat{y}_H) + B\theta_H - \hat{U}_H - t \right] - \hat{\mu}_L, \\ q_L \left[w(y_L) + B\theta_L - 2U_L + \hat{U}_L - t \right] - \mu_H + \nu - \tau_L &= q_L \left[w(y^*) + B\theta_L - \hat{U}_L - t \right] + \hat{\mu}_L + \hat{\nu}. \end{aligned}$$

The first equation implies that $w(y^*) - w(\hat{y}_H) \leq 2(U_H - \hat{U}_H)$, hence $U_L - \hat{U}_L > 0$ by (D.27). Thus $U_L > \bar{U}$, implying $\nu = 0$. It then follows from the second equation above that $w(y_L) + B\theta_L - 2U_L + \hat{U}_L \geq w(y^*) + B\theta_L - \hat{U}_L$, hence $2(U_L - \hat{U}_L) \leq w(y_L) - w(y^*) \leq 0$, which contradicts $U_L > \hat{U}_L$.

• *Subcase A2.* Equation (D.26) is therefore an equality, implying that $y_L = \hat{y}_H = y^* = y_H$ (and $\psi = 0$). Thus $U_H - U_L = y_H \Delta\theta$ and $y_L = y^*$, implying that $\mathcal{C} \in S_H$, so it must coincide with $\widehat{\mathcal{C}}$. Note that $\widehat{\mathcal{C}} \in S_H \cap S_L$ can only occur at $t = t_2$.

Case B: t lies in Region III, so that $\widehat{\mathcal{C}} \in S_L$. If $\mathcal{C} \in S_L$ as well, they must coincide. If $\mathcal{C} \in S_H$ then $y_L = y^*$, $\mu_H = 0$ and $U_H - U_L = y_H \Delta\theta \ \geq \hat{y}_L \Delta\theta = \hat{U}_H - \hat{U}_L$. Therefore:

$$U_H - \hat{U}_H \ \geq U_L - \hat{U}_L = U_L - \bar{U} \geq 0. \tag{D.28}$$

Subtracting from (D.15) its counterpart for $\widehat{\mathcal{C}}$, we now have:

$$\begin{aligned} q_H \left[w(y_H) + B\theta_H - 2U_H + \hat{U}_H - t \right] - \mu_L - \tau_H &= q_H \left[w(y^*) + B\theta_H - \hat{U}_H - t \right] + \hat{\mu}_H \Rightarrow \\ w(y_H) - w(y^*) &\geq 2(U_H - \hat{U}_H), \end{aligned}$$

which together with (D.28) requires that $U_H = \hat{U}_H, U_L = \hat{U}_L$ and $y_H = y^* = \hat{y}_L$, so that $\mathcal{C} = \widehat{\mathcal{C}}$. Here again it must be that $t = t_2$, which corresponds to the only intersection of S_H and S_L. ■

REFERENCES

Acemoglu, D., Kremer, M. and A. Mian (2007) "Incentives in Markets, Firms and Governments," *The Journal of Law, Economics, and Organization,* 24(2), 273-306.

Acharya, V., Gabarro, M., and P. Volpin (2012) "Competition for Managerial Talent, Corporate Governance and Incentive Compensation," CEPR W.P. 8936.

Acharya, V., Pagano, P. and P. Volpin (2012) "Seeking Alpha: Excess Risk Taking and Competition for Managerial Talent," NUY mimeo, May.

Akerlof, G. and and Kranton, R. (2005) "Identity and the Economics of Organizations," *Journal of Economic Perspectives,* 19(1), 9-32.

Armstrong, M. and J. Vickers (2001) "Competitive Price Discrimination", *Rand Journal of Economics,* 32(4), 579-605.

Baker, G., R. Gibbons and K.J. Murphy (1994) "Subjective Performance Measures in Optimal Incentive Contracts", *Quarterly Journal of Economics,* 109, 1125-56.

Bandiera, O., Barankay, I. and I. Rasul (2007) "Incentives for Managers and Inequality among Workers: Evidence from a Firm-Level Experiment", *The Quarterly Journal of Economics,* 122, 729-773.

Baker, G. (1992) "Incentive Contracts and Performance Measurement," *Journal of Political Economy,* 100(3), 98-614.

Bartling, B., Fehr, E. and K. Schmidt (2012) "Screening, Competition and Job Design", *American Economic Review,* forthcoming.

Baumol, W. J. (1990): "Entrepreneurship: Productive, Unproductive, and Destructive," *Journal of Political Economy,* 98, 893-921.

Bebchuk, L. and J. Fried (2004) *Pay without Performance: The Unfulfilled Promise of Executive Compensation.* Cambridge: Harvard University Press, 2004.

Bénabou, R., and J. Tirole (2003), "Intrinsic and Extrinsic Motivation," *Review of Economic Studies,* 70, 489-520.

Bénabou, R., and J. Tirole (2006), "Incentives and Prosocial Behavior," *American Economic Review*, 96(5), 1652-1678.

Bénabou, R., and J. Tirole (2011), "Laws and Norms," NBER Working Paper 17579, November.

Bénabou and Tirole (2013) "Competitive Pay, Work Ethics, and Corporate Image," mimeo.

Bertrand, M. and S. Mullainathan (2001) "Are CEOs Rewarded for Luck? The Ones without Principals Are", *Quarterly Journal of Economics,* 116, 901-932.

Besley, T., and M. Ghatak (2005) "Competition and Incentives with Motivated Agents," *American Economic Review*, 95(3), 616-636.

Besley, T., and M. Ghatak (2006) "Sorting with Motivated Agents: Implications for School Competition and Teacher Incentives," *Journal of the European Economics Association*, 4, 404-414.

Besley, T. and M. Ghatak (2013) "Bailouts and the Optimal Taxation of Bonus Pay", *American Economic Review Papers and Proceedings*, forthcoming.

Bolton, P., Scheinkman, J. and W. Xiong (2006)'"Executive Compensation and Short-Termist Behaviour in Speculative Markets," *Review of Economic Studies,* 73, 577-610.

Brekke, K. A., and K. Nyborg (2008) "Attracting Responsible Employees: Green Production as Labor Market Screening," *Resource and Energy Economics,* 39, 509-526.

Chassagnon, A. and P.A. Chiappori (1997) "Insurance under Moral Hazard and Adverse Selection: the Case of Pure Competition, DELTA mimeo, March.

Crocker, K. and A. Snow (1985) "The Efficiency of Competitive Equilibria in Insurance Markets with Asymmetric Information", *Journal of Public Economics,* 26, 207-219.

Desai, M. (2012) "The Incentive Bubble," *Harvard Business Review*, 90(3), 124-132,

Dewatripont, M., Jewitt, I. and J. Tirole (1999a) "The Economics of Career Concerns, Part 1: Comparing Information Structures", *Review of Economic Studies,* 66, 183-198.

Fabbri, F. and D. Marin (2012) "What explains the Rise in CEO Pay in Germany? A Panel Data Analysis for 1977-2009", CESifo WW.P. 3757.

Fehr, E. and K. Schmidt (2004) "Fairness and Incentives in a Multi-Task Principal-Agent Model," *Scandinavian Journal of Economics,* 106, 453-474.

Frydman, C. (2007) "Rising Through the Ranks: The Evolution of the Market for Corporate Executives, 1936-2003," MIT mimeo, September.

Frydman, C. and D. Jenter (2010) "CEO Compensation," *Annual Review of Financial Economics,* 2(1), 75-102.

Frydman, C. and R. E. Saks (2010) "Executive Compensation: A New View from a Long-Term Perspective, 1936-2005," *Review of Financial Studies,* 23, 2099-2138.

Gabaix, X. and A. Landier (2008) "Why has CEO Pay Increased So Much?," *Quarterly Journal of Economics,* 123: 49-100.

Gawande, A. (2009) "The Cost Conundrum: What a Texas Town Can Teach Us About Health Care," *The New Yorker,* June 1.

Guerrieri, V., Shimer, R. and R. Wright (2010) "Adverse Selection in Competitive Search Equilibrium", *Econometrica,* 78, 1823-1862.

Holmström B. (1982) "Moral Hazard in Teams," *The Bell Journal of Economics,* 13(2), 324-340.

Holmström B., and P. Milgrom (1991) "Multi-Task Principal-Agent Analyzes: Incentive Contracts, Asset Ownership, and Job Design," *Journal of Law, Economics and Organization*, 7, 24-52.

Itoh, H. (1991) "Incentives to Help in Multi-Agent Situations", *Econometrica,* 59, 611-636.

Kosfeld, M. and F. von Siemens (2011) "Competition, Cooperation and Corporate Culture", *Rand Journal of Economics,* 42, 23-43.

Laffont, J.J., and J. Tirole (1986), "Using Cost Observation to Regulate Firms," *Journal of Political Economy*, 94, 614-641.

Lazear, E. (2000) "Performance Pay and Productivity," *American Economic Review,* 90(5), 1346-1361.

Lemieux, T., MacLeod, B. and D. Parent (2009) "Performance Pay and Wage Inequality," *Quarterly Journal of Economics,* 104 (1), 1-49.

Marin, D. and T. Verdier (2009) "Globalization and the Empowerment of Talent", *Journal of International Economics,* 86, 209-223.

Maskin, E., and J. Tirole (1992), "The Principal-Agent Relationship with an Informed Principal, II: Common Values," *Econometrica*, 60(1), 1-42.

Netzer, N. and F. Scheuer (2010) "Competitive Markets without Commitment," *Journal of Political Economy,* 118,(6), 1079-1109

Philippon, T. and A. Reshef (2012) "Wages and Human Capital in the U.S. Financial Industry: 1909-2006", *Quarterly Journal of Economics,* forthcoming.

Prendergast, C. (1992) "Career Development and Specific Human Capital Collection," *Journal of the Japanese and International Economies, 6, 207-227.*

Prendergast, C. (2007) "The Motivation and Bias of Bureaucrats," *American Economic Review,* 97(1), 180–196.

Raith, M. (2003) "Competition, Risk, and Managerial Incentives," *American Economic Review,* 93(4), 1425–1436.

Ramalingam, A. and M. Rauh (2010) "The Firm as a Socialization Device," *Management Science,* 56(12), 2191-2206.

Rochet, J.C. and L. Stole (2002) "Nonlinear Pricing with Random Participation," *Review of Economic Studies,* 69, 1, 277-311.

Rosen, S. (1981) "The Economics of Superstars", *American Economic Review,* 71: 845-858.

Rothschild, M., and J. Stiglitz (1976) "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," *The Quarterly Journal of Economics*, 90(4), 629–649.

Stantcheva, S. (2012) "Optimal Income Taxation with Adverse Selection in the Labor Market", mimeo, MIT.

Schmidt, K. (1997) "Managerial Incentives and Product Market Competition," *Review of Economic Studies,* 64(2), 191-213.

Shearer, B. (2004) "Piece Rates, Fixed Wages and Incentives: Evidence from a Field Experiment," *Review of Economic Studies,* 71, 513-534.

Stewart, J. (1994) "The Welfare Implications of Moral Hazard and Adverse Selection in Competitive Insurance Markets, *Economic Inquiry,* 32(2), 193-208.

Veiga, A. and G. Weyl (2012) "Multidimensional Product Design," University of Chicago mimeo, May.

Villas-Boas, M. and U. Schmidt-Mohr (1999) "Oligopoly with Asymmetric Information: Differentiation in Credit Markets," *The RAND Journal of Economics,* 30(3), 375-396.